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Study Guide

Direct, Inverse, and Joint Variation

A **direct variation** can be described by the equation $y = kx^n$. The k in this equation is called the **constant of variation.** To express a direct variation, we say that y varies directly as x^n. An inverse variation can be described by the equation $y = \frac{k}{x^n}$ or $x^n y = k$. When quantities are **inversely proportional,** we say they *vary inversely* with each other. **Joint variation** occurs when one quantity varies directly as the product of two or more other quantities and can be described by the equation $y = kx^n z^n$.

Example 1   **Suppose y varies directly as x and $y = 14$ when $x = 8$.**
       a. Find the constant of variation and write an equation of the form $y = kx^n$.
       b. Use the equation to find the value of y when $x = 4$.

       a. The power of x is 1, so the direct variation equation is $y = kx$.

$$y = kx$$
$$14 = k(8) \quad y = 14, \, x = 8$$
$$1.75 = k \quad \text{Divide each side by 8.}$$

       The constant of variation is 1.75. The equation relating x and y is $y = 1.75x$.

       b. $y = 1.75x$
$$y = 1.75(4) \quad x = 4$$
$$y = 7$$

       When $x = 4$, the value of y is 7.

Example 2   **If y varies inversely as x and $y = 102$ when $x = 7$, find x when $y = 12$.**

       Use a proportion that relates the values.

$$\frac{x_1{}^n}{y_2} = \frac{x_2{}^n}{y_1}$$

$$\frac{7}{12} = \frac{x}{102} \qquad \text{Substitute the known values.}$$

$$12x = 714 \qquad \text{Cross multiply.}$$

$$x = \frac{714}{12} \text{ or } 59.5 \quad \text{Divide each side by 12.}$$

       When $y = 12$, the value of x is 59.5.

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Practice

Direct, Inverse, and Joint Variation

Write a statement of variation relating the variables of each equation.
Then name the constant of variation.

1. $-\dfrac{x^2}{y} = 3$

2. $E = IR$

3. $y = 2x$

4. $d = 6t^2$

Find the constant of variation for each relation and use it to write an
equation for each statement. Then solve the equation.

5. Suppose y varies directly as x and $y = 35$ when $x = 5$. Find y when $x = 7$.

6. If y varies directly as the cube of x and $y = 3$ when $x = 2$, find x when $y = 24$.

7. If y varies inversely as x and $y = 3$ when $x = 25$, find x when $y = 10$.

8. Suppose y varies jointly as x and z, and $y = 64$ when $x = 4$ and $z = 8$.
   Find y when $x = 7$ and $z = 11$.

9. Suppose V varies jointly as h and the square of r, and $V = 45\pi$ when $r = 3$ and
   $h = 5$. Find r when $V = 175\pi$ and $h = 7$.

10. If y varies directly as x and inversely as the square of z, and $y = -5$ when $x = 10$ and
   $z = 2$, find y when $x = 5$ and $z = 5$.

11. *Finances* Enrique deposited \$200.00 into a savings account. The simple
   interest I on his account varies jointly as the time t in years and the principal P.
   After one quarter (three months), the interest on Enrique's account is \$2.75.
   Write an equation relating interest, principal, and time. Find the constant of
   variation. Then find the interest after three quarters.