Key

Math III Trigonometry Review

F.TF.1, 2, 5, & 8

Trig Ratios

$Sin\theta = \frac{Opposite}{Hypotenuse}$ $\quad Csc\theta = \frac{Hypotenuse}{Opposite}$

$Cos\theta = \frac{Adjacent}{Hypotenuse}$ $\quad Sec\theta = \frac{Hypotenuse}{Adjacent}$

$Tan\theta = \frac{Opposite}{Adjacent}$ $\quad Cot\theta = \frac{Adjacent}{Opposite}$

Trig Identities

$Tan\theta = \frac{sin\theta}{cos\theta}$ $\quad Cot\theta = \frac{cos\theta}{sin\theta}$

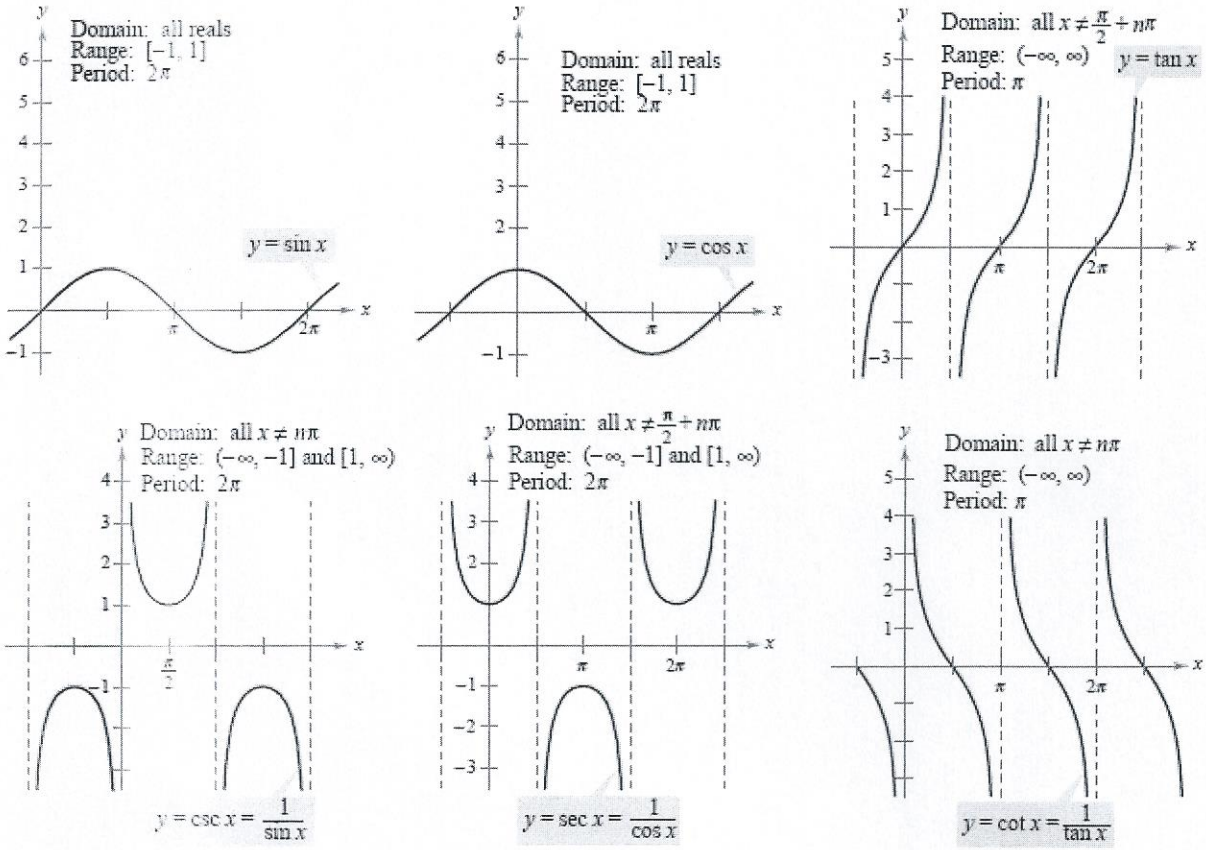
$\csc\theta = \frac{1}{\sin\theta}$ $\quad \sin\theta = \frac{1}{\csc\theta}$

$\sec\theta = \frac{1}{\cos\theta}$ $\quad \cos\theta = \frac{1}{\sec\theta}$

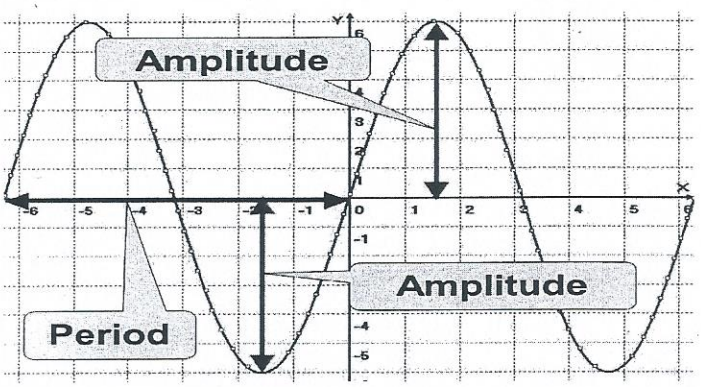
$\cot\theta = \frac{1}{\tan\theta}$ $\quad \tan\theta = \frac{1}{\cot\theta}$

$\sin^2\theta + \cos^2\theta = 1$

Trig Graphs



The graphs of the six trigonometric functions



Amplitude - half of the distance from the maximum and minimum

Period – The horizontal length of one complete cycle

Frequency – The number of cycles the function completes in a given interval.

Midline – The horizontal line half way between the maximum and minimum.

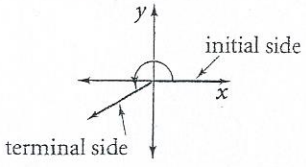
Graph Trigonometric Functions

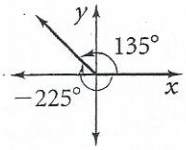
$$y = a \sin b(\theta - h) + k$$

amplitude: a — period: b — phase shift: h — vertical shift: k

Coterminal angles – two angles in standard position that have the same terminal side.

Standard Position





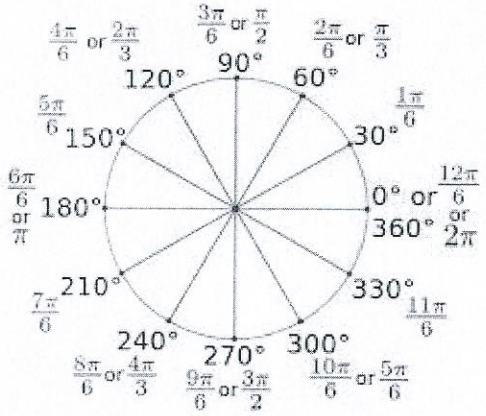
Angles that have measures 135° and −225° are coterminal.

1 radian ≈ 57^0

To change from degrees to radians – multiply the degrees by $\frac{\pi}{180}$

To change from radians to degrees – multiply the radian by $\frac{180}{\pi}$

Unit Circle



Review Problems

1. Which expression is equivalent to $\sin\theta \cos\theta \csc\theta$?
   A. $\sin\theta$ (B.) $\cos\theta$ C. $\sec\theta$ D. $\tan\theta$

$= \sin\theta \cos\theta \frac{1}{\sin\theta} = \cos\theta$

2. Which expression is equivalent to $\cos\theta + \tan\theta \sin\theta$?
   (A.) $\sec\theta$ B. $\tan\theta$ C. $\sin\theta$ D. $\cos\theta$

$= \cos\theta + \frac{\sin\theta}{\cos\theta} \cdot \sin\theta = \frac{\cos\theta}{1} \cdot \frac{\cos\theta}{\cos\theta} + \frac{\sin^2\theta}{\cos\theta}$

$= \frac{\cos^2\theta}{\cos\theta} + \frac{\sin^2\theta}{\cos\theta}$

$= \frac{\cos^2\theta + \sin^2\theta}{\cos\theta}$

$= \frac{1}{\cos\theta} = \sec\theta$

3. Which expression is equivalent to $\frac{\cos\theta}{1-\sin\theta} - \tan\theta$?
   (A.) $\sec\theta$ B. $\sin\theta$ C. $\cos\theta$ D. $\csc\theta$

$\frac{\cos\theta}{1-\sin\theta} - \tan\theta$

$\frac{\cos\theta}{1-\sin\theta} \cdot \frac{\cos\theta}{\cos\theta} - \frac{\sin\theta}{\cos\theta} \cdot \frac{(1-\sin\theta)}{(1-\sin\theta)}$

$\frac{\cos^2\theta}{\cos\theta(1-\sin\theta)} - \frac{\sin\theta - \sin^2\theta}{\cos\theta(1-\sin\theta)}$

$\frac{\cos^2\theta + \sin^2\theta - \sin\theta}{\cos\theta(1-\sin\theta)} = \frac{1-\sin\theta}{\cos\theta(1-\sin\theta)} = \frac{1}{\cos\theta} = \sec\theta$

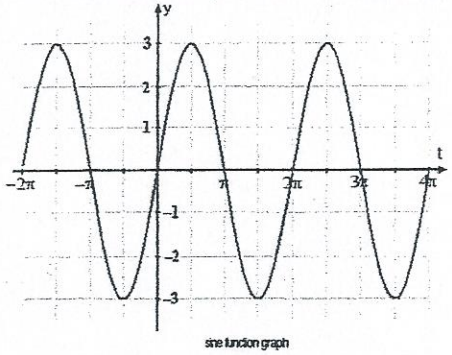
4. William put the tip of his pencil on the outer edge of a graph of the unit circle at the point (0, -1). He moved this pencil tip through an angle of $\frac{4\pi}{3}$ radians in the counterclockwise direction along the edge of the circle. At what angle of the unit circle did William's pencil tip stop?

A. $\frac{\pi}{3}$ (B.) $\frac{5\pi}{6}$ C. $\frac{7\pi}{6}$ D. $\frac{5\pi}{3}$

$\frac{4\pi}{3}$ radians $= 240^\circ$

So if we start at (0,-1) and rotate 240° counterclockwise we land on $\frac{5\pi}{6}$

5. Which of the following functions is graphed below?



sine function graph

amplitude = 3

A. $y = 3\cos\theta$ (B.) $y = 3\sin\theta$ C. $y = \cos 3\theta$ D. $y = \sin 3\theta$

6. A Ferris wheel has a diameter of 114 feet and is 5 feet off the ground. After a person gets on the bottom car, the Ferris wheel rotates 170° counterclockwise before stopping. How high above the ground is the car when it has stopped?

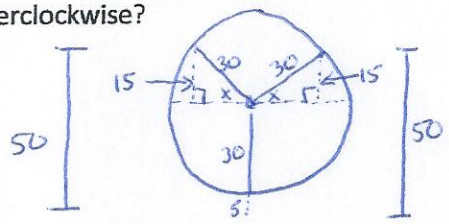
A. 56 feet B. 62 feet C. 80 feet (D.) 118 feet

$\sin(80) = \frac{x}{57}$

$x = 56.13$

$57 + 56.13 + 5 = 118.13$

7. A Ferris wheel has a radius of 30 meters and is 5 meters off the ground. If a person on the Ferris wheel is 50 meters above the ground, at what degree(s) had the Ferris wheel rotated counterclockwise?



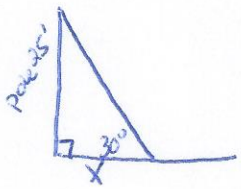
$\sin(x) = \frac{15}{30}$

$\sin^{-1}\left(\frac{15}{30}\right) = 30$

So either 120° or 240°

8. A rope is attached to the top of a 25-foot pole. The pole is perpendicular to the ground. Approximately how far from the base of the pole must the rope be attached to make a 30° angle with the ground?

A. 12.5 feet B. 14.4 feet (C.) 43.3 feet D. 50.0 feet

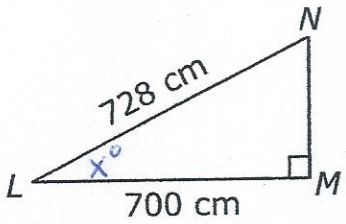


$\tan(30) = \frac{25}{x}$

$x \cdot \tan(30) = 25$

$x = \frac{25}{\tan(30)} = 43.3$

9. In right triangle *LMN*, *LN* = 728 cm and *LM* = 700 cm.



What is the ***approximate*** measure of ∠*NLM*?

(A) 15.9°

B 16.6°

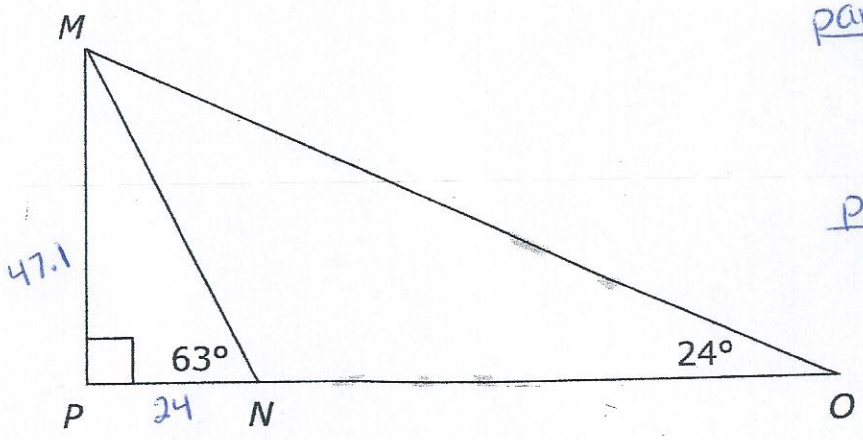
C 73.4°

D 74.1°

$\cos(x) = \frac{700}{728}$

$\cos^{-1}\left(\frac{700}{728}\right) = 15.9$

10. In the diagram below, Triangle *MPO* is a right triangle and $\overline{PN}$ = 24 ft.



- What is the length of $\overline{MP}$?
- How much longer is $\overline{MO}$ than $\overline{NM}$?
- How far is point *O* from point *N*?

part 1 $\tan(63) = \frac{x}{24}$

$x = 24 \cdot \tan(63)$

$x = 47.1$ $\overline{MP} = 47.1$

part 2

$\overline{MN}$ $\cos(63) = \frac{24}{x}$

$x \cdot \cos(63) = 24$

$x = \frac{24}{\cos(63)} = 52.86$

$\overline{MN} = 52.86$

$\overline{MO}$ $\sin(24) = \frac{47.1}{x}$

$x \cdot \sin(24) = 47.1$

$x = 115.8$

$\overline{MO} = 115.8$

$115.8 - 52.86 = 62.94$

part 3

$\tan(24) = \frac{47.1}{x}$

$x \cdot \tan(24) = 47.1$

$x = \frac{47.1}{\tan(24)} = 105.79$

$105.79 - 24 = 81.79$

11. Which expression is equivalent to $\frac{sin^4(\theta)-cos^4(\theta)}{sin^2(\theta)-sin^2(\theta)}$, where $sin^2(\theta) \neq cos^2(\theta)$?

A. $sin^2(\theta) - cos^2(\theta)$

B. $cos^2(\theta) - sin^2(\theta)$

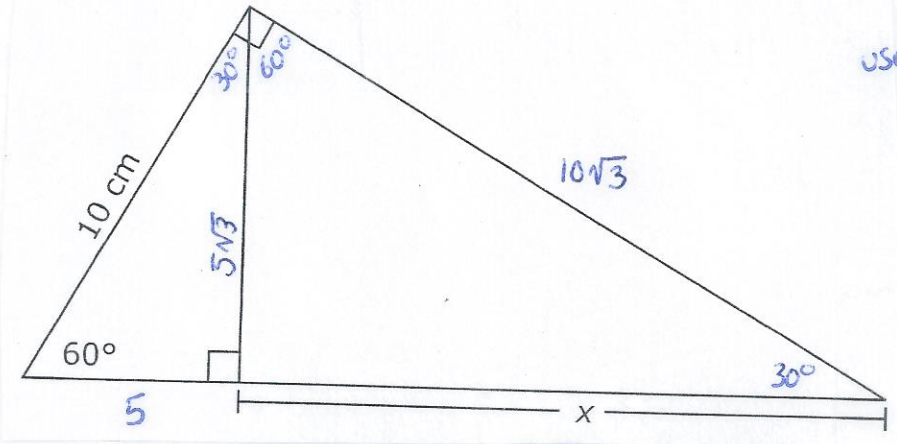
C. 2

(D.) 1

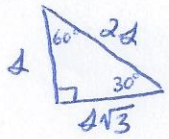
$\frac{(\sin^2\theta+\cos^2\theta)(\sin^2\theta-\cos^2\theta)}{\sin^2\theta-\sin^2\theta} = \frac{\sin^2\theta+\cos^2\theta}{1}$

$= \frac{1}{1} = 1$

12. What is the value of x in the triangle below?



use 30-60-90 rule



$5\sqrt{3}\sqrt{3}$

$5\sqrt{9}$

$5(3) = 15$

A. $\frac{5\sqrt{3}}{2}$ cm B. $5\sqrt{3}$ cm C. 10 cm (D.) 15 cm

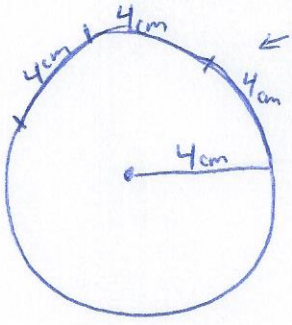
13. Which angle, in standard position, is NOT coterminal with the others?

(A.) -190^0 B. -170^0 C. 190^0 D. 550^0

$r = 4$

14. The diameter of a circle is 8 centimeters. A central angle of the circle intercepts an arc of 12 centimeters. What is the radian measure of the angle?

A. $\frac{3}{2}$ (B.) 3 C. 4 D. 8π



← takes 3 radians to get to 12cm

15. In a circle, an arc of length 8π cm is intercepted by a central angle of $\frac{2\pi}{3}$ radians. What is the radius of the circle?

120°

$\frac{120}{360} \cdot 2\pi r = 8\pi$

$\left(\frac{3}{2}\right)\frac{2\pi r}{3} = 8\pi\left(\frac{3}{2}\right)$

$\pi r = 12\pi$

$r = 12$

A. $\frac{3\pi}{16}$ cm B. $\frac{16\pi}{3}$ cm C. $\frac{16\pi^2}{3}$ cm D. 12 cm

16. What is the amplitude of $y = 3\sin 4\theta$?

A. $\frac{4}{3}$ B. 3 C. 4 D. 2π

17. Which answer choice describes $y = -\sin 2\theta$?

A. amplitude -1, period 4π B. amplitude 1, period π

C. amplitude 2, period $-\pi$ D. amplitude 2π, period 1

18. Which function has a period of 4π and an amplitude of 8?

A. $y = -8\sin 8\theta$ B. $y = -8\sin\frac{1}{2}\theta$ C. $y = 8\sin 2\theta$ D. $y = 4\sin 8\theta$

19. Which function is a phase shift of $y = \sin\theta$ by 5 units to the left?

A. $y = 5\sin\theta$ B. $y = \sin\theta + 5$ C. $y = \sin(\theta + 5)$ D. $y = \sin 5\theta$