Math III
Quadratics Review

Key

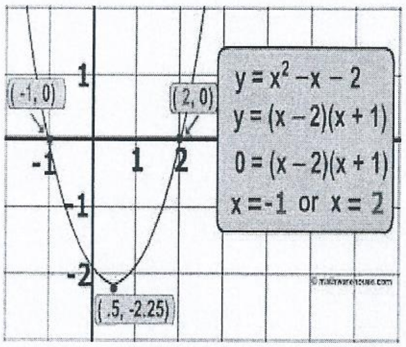
Standard form $f(x) = ax^2 + bx + c$

c is the y-intercept of a quadratic, positive a (faces up like a U), negative a (faces down)

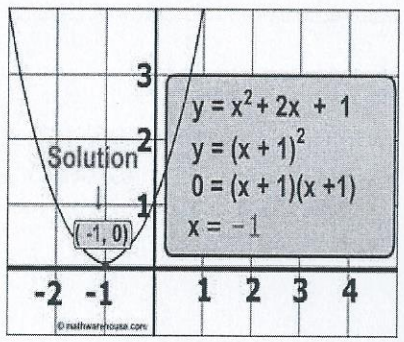
Vertex Form $f(x) = a(x-h)^2 + k \qquad vertex(h,k)$

Solutions (known as x-intercepts, zeroes, or roots) of a quadratic can be found four ways:

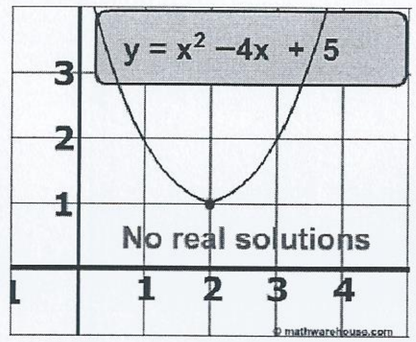
Graphing – Graph the function in y=, 2nd, trace, zero



Two Solutions



One solution



Imaginary Solutions

Factoring - transform a quadratic from standard form into factored

Ex/ Solve $f(x) = 3x^2 + 7x - 6$

Factor $f(x) = (3x-2)(x+3)$

Set each factor equal to zero and solve for *x*

3x - 2 = 0   x + 3 = 0

3x = 2   x = -3

$x = \frac{2}{3}$

Quadratic Formula $x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$, use the same *a, b, c* from standard form

Ex/ $f(x) = 3x^2 + 7x - 6 \qquad a = 3, b = 7, and\ c = -6$

$$x = \frac{-7 \pm \sqrt{(7)^2 - 4(3)(-6)}}{2(3)}$$

$$= \frac{-7 \pm 11}{6}$$

So $x = \frac{2}{3}$ and x = -3

Completing the Square *if $a \neq 1$, divide it from the equation

Ex/ $x^2 - 6x + 25 = 0$

$x^2 - 6x \quad = -25$ *move the c value to the other side*

$x^2 - 6x + 9 = -25 + 9$ *add* $\left(\frac{b}{2}\right)^2$ *to both sides*

$(x-3)^2 = -16$ *factor the left side*

$x - 3 = \pm\sqrt{-16}$ *take the square root to both sides*

$x - 3 = \pm 4i$ *simplify the right side*

$x = 3 \pm 4i$ *solve for x*

Ex/ $3x^2 - 6x - 9 = 0$

$3x^2 - 6x \quad = 9$

$x^2 - 2x \quad = 3$

$x^2 - 2x + 1 = 3 + 1$

$(x-1)^2 = 4$

$x - 1 = \pm 2$

$x = 3, -1$

Factor by grouping

EX/ $8xy - 2x + 20y - 5$

$2x(4y-1)5(4y-1)$ split expression in half so a GCF can be pulled out and find the GCF of each side

$(4y-1)(2x+5)$ Pull out the common factor, the GCF's become a factor together

Imaginary Numbers

i is an imaginary number and occurs when a negative is under a square root

$$i^2 = -1$$

Examples

1. Simplify $\sqrt{-8}$

$i\sqrt{8}$

4 2

2 2

$2i\sqrt{2}$

2. Simplify $\sqrt{-32}$

$i\sqrt{32}$

4 8

2 2 4 2

2 2

$4i\sqrt{2}$

3. Simplify $(-1+5i)+(-2-3i)$

$-3+2i$

4. Simplify $(3+2i)(2-5i)$

$6-15i+4i-10i^2$

$+10$

$16-11i$

5. What are the values of x and y when $(5+4i)-(x+yi)=(-1-3i)$?

A. x = 6, y = 7   C. x = 6, y = i

B. x = 4, y = i D. x = 4, y = 7

6. Which is equivalent to $(4-3i)^2+(6+i)^2$?

A. 30   B. 50   C. $42-12i$ D. $62-12i$

$(4-3i)(4-3i)+(6+i)(6+i)$

$16-24i+\overset{-9}{9i^2}$ $36+12i+\overset{-1}{i^2}$ $42-12i$

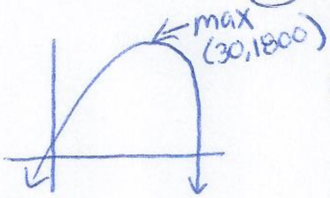
7. For which equation is the x-coordinate of the vertex at 4?

A. $f(x)=x^2-8x+15$ C. $f(x)=x^2+6x+8$

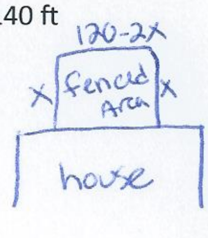
B. $f(x)=-x^2-4x+12$ D. $f(x)=-x^2-2x+2$

8. Adrian is using 120 feet of fencing to enclose a rectangular area for her puppy. One side of the enclosure will be her house. The function $f(x)=x(120-2x)$ represents the area of the enclosure. What is the greatest area that Adrian can enclose for fencing?

A. 1650 ft   B. 1800 ft   C. 1980 ft   D. 2140 ft



plug in $y=x(120-2x)$
find max area



If 5-3i is a solution then so is 5+3i

9. If $5 - 3i$ is a solution for $x^2 + ax + b = 0$, where a and b are real numbers, what is the value of b?

A. 10 B. 14 C. 34 D. 40

$(x-(5-3i))(x-(5+3i))$

$(x-5+3i)(x-5-3i)$

$=x^2-5x-\cancel{3xi}-5x+25+\cancel{15i}+\cancel{3xi}-\cancel{15i}-\overset{+9}{\cancel{9i^2}}$

$=x^2-10x+34$

10. The graph of the function x^3 will be shifted down 2 units and to the right 3 units. Which is the function that corresponds to the resulting graph?

A. $g(x) = (x + 3)^2 + 2$ C. $g(x) = (x + 3)^2 - 2$

B. $g(x) = (x - 3)^2 + 2$ D. $g(x) = (x - 3)^2 - 2$

$y=a(x-h)^2+K$

$-h$ right $-K$ down

$+h$ left $+K$ up

11. Which choice shows the solutions to the function $8x^2 + 3x = -7$?

A. $\frac{-3\pm i\sqrt{215}}{16}$ B. $\frac{3\pm i\sqrt{215}}{16}$ C. $\frac{-3\pm\sqrt{233}}{16}$ D. $\frac{3\pm\sqrt{233}}{16}$

$\underset{a}{8x^2}+\underset{b}{3x}+\underset{c}{7}=0$

$\frac{-3\pm\sqrt{(3)^2-4(8)(7)}}{2(8)}$

$=\frac{-3\pm\sqrt{-215}}{16}=\frac{-3\pm i\sqrt{215}}{16}$

12. What value of h is needed to complete the square for the equation

$$x^2 + 10x - 8 = (x - h)^2 - 33$$

A. -25 B. -5 C. 5 D. 25

$x^2+10x-8$

$x^2+10x+25=8+25$

$(x+5)^2=33$

$(x+5)^2-33$

13. The equation $2x^2 - 5x = -12$ is rewritten in the form of $2(x - p)^2 + q = 0$. What is the value of q?

A. $\frac{167}{16}$ B. $\frac{71}{8}$ C. $\frac{25}{8}$ D. $\frac{25}{16}$

$(2x^2-5x+\_\_)+12=0$

$2(x^2-\frac{5}{2}x+\frac{25}{16})+12-\frac{25}{8}$

$(\frac{96}{8})$

$2(x-\frac{5}{4})^2+\frac{71}{8}=0$

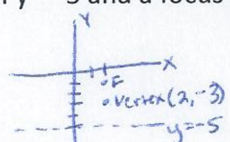
*The vertex is halfway between the focus and directrix

*y value of focus = y value of vertex $+\frac{1}{4a}$

14. Which is an equation of a parabola what has a directrix of y = -5 and a focus at (2, -1)?

A. $y = \frac{1}{2}(x + 2)^3 + 2$ C. $y = \frac{1}{8}(x + 2)^3 + 3$

B. $y = \frac{1}{8}(x - 2)^3 - 3$ D. $y = \frac{1}{2}(x - 2)^3 - 2$

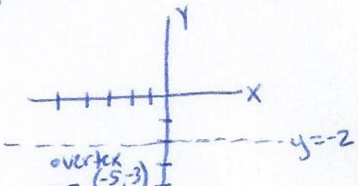


$-1=-3+\frac{1}{4a}$

$\frac{2}{1}=\frac{1}{4a}$

$\frac{1}{8}=\frac{8a}{8}$ $a=1/8$

15. Find the equation of a parabola with a focus of (-5, -4) and a directrix of y = -2



vertex (-5,-3)

$y=-\frac{1}{4}(x+5)^2-3$

$-4=-3+\frac{1}{4a}$

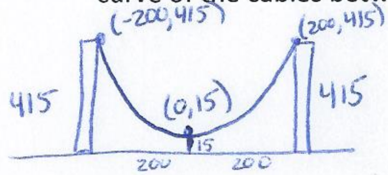
$+3$ $+3$

$\frac{-1}{1}=\frac{1}{4a}$

$\frac{1}{-4}=\frac{-4a}{-4}$

$a=-\frac{1}{4}$

16. A suspended bridge has two cables secured at either end of the span by two supporting towers. The cables are attached to the tops of the towers. In the section between the two towers, the cables form a parabolic curve. At their lowest point, the cables are 15 feet from the surface of the bridge. The towers are 400 feet apart, and the vertical distance from the surface of the bridge to the top of each tower is 415 feet. What is the quadratic equation that describes the curve of the cables between the towers? Use the lowest point as the y-intercept.



Can use QuadReg or by hand

$\boxed{\text{Stat}}$ $\boxed{\text{edit}}$ $\boxed{\text{Stat}}$

$\boxed{\text{calc}}$ $\boxed{\text{quad reg}}$

$a = .01$
$b = 0$
$c = 15$

$y = .01x^2 + 15$

or

$y = \frac{1}{100}x^2 + 15$

$y = a(x-h)^2 + k$

$415 = a(200-0)^2 + 15$

$415 = 40{,}000a + 15$

$400 = 40{,}000a$

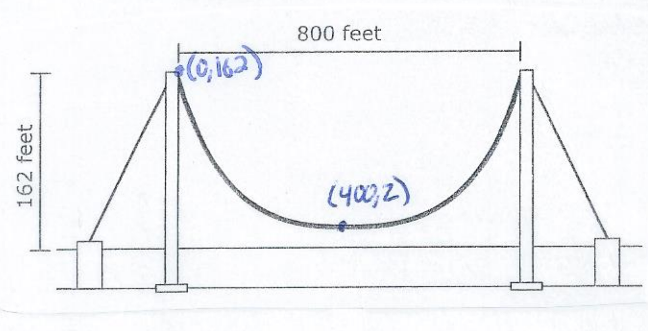
$\frac{400}{40{,}000} = \frac{40{,}000a}{40{,}000}$

$a = \frac{400}{40{,}000} = \frac{1}{100}$

$y = \frac{1}{100}x^2 + 15$

17. The towers of a suspended bridge are 800 feet apart and rise 162 feet higher than the road. Suppose that the cable between the towers has the shape of a parabola and is 2 feet higher than the road at the point halfway between the towers.

Can use y axis on a tower or the lowest point. I used the first tower as y.



$y = a(x-h)^2 + k$

$162 = a(0-400)^2 + 2$

$162 = 160{,}000a + 2$

$160 = 160{,}000a$

$\frac{160}{160{,}000} = \frac{160{,}000a}{160{,}000}$

$a = \frac{1}{1000}$

$y = \frac{1}{1000}(x-400)^2 + 2$

plug in $x = 120$

$y = \frac{1}{1000}(120-400)^2 + 2$

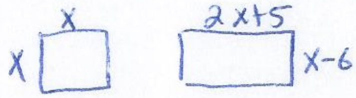
$y = 80.4$

← Since my left tower is at $x=0$ then 120' from the tower is $x = 120$

What is the approximate height of the cable 120 feet from either tower?

A. 80 feet B. 74 feet C. 22 feet D. 16 feet

18. A square and a rectangle have the same area. The length of the rectangle is five inches more than twice the length of the side of the square. The width of the rectangle is 6 inches less than the side of the square. Find the length of the side of the square.



$\frac{\text{Area}}{L \cdot W} = \frac{\text{Area}}{L \cdot W}$

$x^2 = (x-6)(2x+5)$

$x^2 = 2x^2 - 7x - 30$

$-x^2 \quad -x^2$

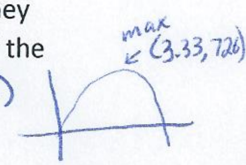
$0 = x^2 - 7x - 30$

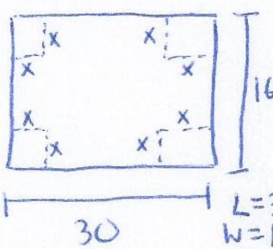
$(x+3)(x-10)$

$\cancel{x=-3}$ $\underline{x = 10}$

can't happen

19. A cardboard box company has been contracted to manufacture open-top rectangular storage boxes for a manufacturing company. The company has 30 cm X 16 cm cardboard sheets. They plan to cut a square from each corner of the sheet and bend up the sides to form the box. If the company wants to make boxes with the largest possible volume: $y = (30-2x)(16-2x)(x)$ graph





- Find the dimensions of the square being cut out.
  $x = 3.33$ 3.33 by 3.33
- What are the dimensions of the box? $L = 30-2x = 30-2(3.33) = 23.34$ cm $W = 16-2x = 16-2(3.33) = 9.34$ cm $H = 3.33$ cm
- What is the maximum volume of the box?
  $726 cm^3$

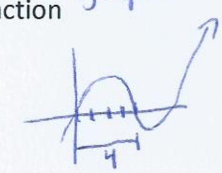
$L = 30-2x$
$W = 16-2x$
$H = x$

20. A box with an open top will be constructed from a rectangular piece of cardboard.
    - The piece of cardboard is 8 inches by wide and 12 inches long.
    - The box will be constructed by cutting out equal squares of sides x at each corner and then folded up at the sides.

$L = 12-2x$
$W = 8-2x$
$H = x$
$y = (12-2x)(8-2x)(x)$
graph

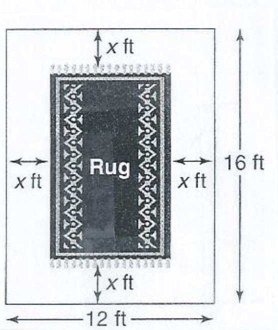
What is the entire domain for the function $V(x)$ that gives the volume of the box as a function of x?

A. $0 < x < 4$ B. $0 < x < 6$ C. $0 < x < 8$ D. $0 < x < 12$

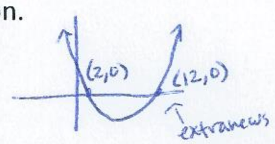


21. Abey Numkena is an interior designer. She has been asked to locate an oriental rug for a new corporate office. As a rule, the rug should cover $\frac{1}{2}$ of the total floor area with a uniform width surrounding the rug.

$16 \cdot 12 = 192$ rug $= \frac{1}{2}(192) = 96$



- If the dimensions of the room are 12 feet by 16 feet, write an equation to model the situation. $y = (12-2x)(16-2x) - 96$ or $y = 4x^2 - 56x + 96$
- Sketch a graph of the function.



$x = 2$

- What are the dimensions of the rug?
  $L = (16-2x) = 16-2(2) = 12$
  $W = (12-2x) = 12-2(2) = 8$
  8 X 12