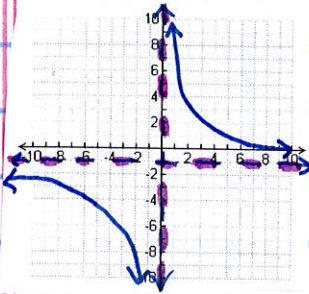Unit 7 Review

1. $y = \dfrac{8}{x} - 1$

   $x = 0 \quad y = -1$

   Domain: $\mathbb{R} - 0$

   Range: $\mathbb{R} - [-1]$
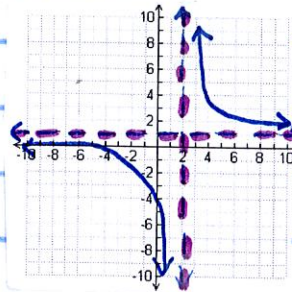

2. $y = \dfrac{1}{x-2} + 1$

   $x = 2 \quad y = 1$

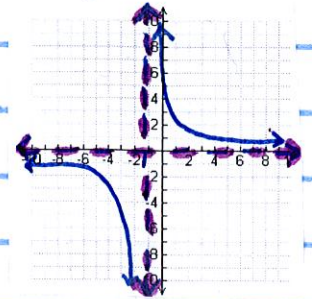   Domain: $\mathbb{R} - [2]$

   Range: $\mathbb{R} - [1]$

3. $y = \dfrac{2}{x+1}$

   $x = -1 \quad y = 0$

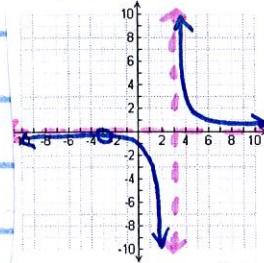   Domain: $\mathbb{R} - [-1]$

   Range: $\mathbb{R} - [0]$

4. $y = \dfrac{x+3}{(x^2-9)} = \dfrac{x+3}{(x-3)(x+3)} = \boxed{\dfrac{1}{x-3}}$

   P.O.D: $x + 3 = 0$
   $x = -3$

   Vertical Asy: $x - 3 = 0$
   $x = 3$

   Horizontal Asy: $\dfrac{x}{x^2} \quad y = 0$

   Domain: $\mathbb{R} - [-3, 3]$
   Range: $\mathbb{R} - [0]$
   x-int: None
   y-int: $(0, -1/3)$

5. $y = \dfrac{5-x}{x^2-25} = \dfrac{-x+5}{x^2-25} = \dfrac{-(x-5)}{(x+5)(x-5)} = \boxed{\dfrac{-1}{x+5}}$

   P.O.D: $x - 5 = 0$
   $x = 5$

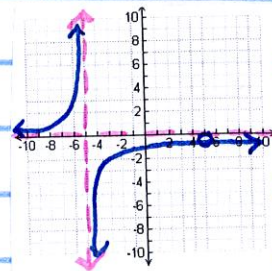   Vertical Asy: $x + 5 = 0$
   $x = -5$

   Horizontal Asy: $\dfrac{x}{x^2} \quad y = 0$

   Domain: $\mathbb{R} - [5, -5]$
   Range: $\mathbb{R} - [0]$
   x-int: None
   y-int: $(0, -1/5)$

6. $\dfrac{5x^2y}{10xy^4} = \dfrac{5 \cdot x \cdot x \cdot y}{10 \cdot x \cdot y \cdot y \cdot y \cdot y} = \boxed{\dfrac{x}{2y^3}} \quad x \neq 0 \ y \neq 0$

7. $\dfrac{4d^2 + 8d}{2d} = \dfrac{4d(d+2)}{2d} = \boxed{2(d+2) \quad d \neq 0}$

8. $\dfrac{x^2 + 9x + 18}{x+6} = \dfrac{(x+3)(x+6)}{x+6} = \boxed{x+3 \quad x \neq -6}$

9. $\dfrac{x^2 - 2x - 8}{x+3} \div \dfrac{x-4}{x+3} = \dfrac{(x-4)(x+2)}{(x+3)} \cdot \dfrac{x+3}{x-4} = \boxed{x+2 \quad x \neq -3, 4}$

10. $\dfrac{3x+1}{x^2 - x - 6} \div \dfrac{6x^2 + 11x + 3}{x^2 + 4x + 4} = \dfrac{3x+1}{(x-3)(x+2)} \cdot \dfrac{(x+2)(x+2)}{(2x+3)(3x+1)} = \boxed{\dfrac{x+2}{(x-3)(2x+3)} \quad x \neq -2, 3, -\frac{3}{2}, -\frac{1}{3}}$

$\dfrac{6x^2 + 11x + 3}{x^2 + 11x + 18} \quad \dfrac{(x+\frac{3}{2})(x+\frac{1}{3})}{}$

$\dfrac{(x+9)(x+2)}{6 \quad 6} \quad (2x+3)(3x+1)$

11. $\dfrac{3x^4 - x^3 - 2x^2}{6x^2 - 2x - 4} = \dfrac{x^2(3x^2 - x - 2)}{2(3x^2 - x - 2)} = \boxed{\dfrac{x^2}{2}}$

12. $\dfrac{2x^2 + 5x - 3}{x^2 - 4x} \cdot \dfrac{2x^3 - 8x^2}{x^2 + 6x + 9} = \dfrac{(x+3)(2x-1)}{x(x-4)} \cdot \dfrac{2x^2(x-4)}{(x+3)(x+3)} = \boxed{\dfrac{2x(2x-1)}{x+3} \quad x \neq 0, 4, -3}$

$\dfrac{2x^2 + 5x - 3}{x^2 + 5x - 6}$

$\dfrac{(x+6)(x-1)}{2 \quad 2} \to (x+3)(2x-1)$

13. $\dfrac{x^2+3x+2}{x-1} \cdot \dfrac{1-x}{x+2} = \dfrac{(x+2)(x+1)}{x-1} \cdot \dfrac{-(x-1)}{x+2} = \boxed{\dfrac{-(x+1)}{x \neq 1, -2}}$

14. $\dfrac{6x+1}{x+2} + \dfrac{2x-5}{2x+4} = \left(\dfrac{2}{2}\right)\dfrac{6x+1}{x+2} + \dfrac{2x-5}{2(x+2)}$ LCD: $2(x+2)$

$\dfrac{12x+1}{2(x+2)} + \dfrac{2x-5}{2(x+2)} = \dfrac{12x+1+2x-5}{2(x+2)} = \dfrac{14x-4}{2(x+2)} = \dfrac{2(7x-2)}{2(x+2)} = \boxed{\dfrac{7x-2}{x+2}}$

15. $\dfrac{8}{x^2-25} + \dfrac{9}{x-5} = \dfrac{8}{(x-5)(x+5)} + \dfrac{9}{x-5}\dfrac{(x+5)}{(x+5)}$ LCD: $(x+5)(x-5)$

$\dfrac{8}{(x-5)(x+5)} + \dfrac{9x+45}{(x-5)(x+5)} = \boxed{\dfrac{9x+53}{(x+5)(x-5)}}$

16. $\dfrac{x-3}{x^2+3x} + \dfrac{7}{x+3} = \dfrac{x-3}{x(x+3)} + \dfrac{7}{x+3}\dfrac{(x)}{(x)}$ LCD: $x(x+3)$

$\dfrac{x-3}{x(x+3)} + \dfrac{7x}{x(x+3)} = \dfrac{x+7x-3}{x(x+3)} = \boxed{\dfrac{8x-3}{x(x+3)}}$

17. $\dfrac{3x}{(x+2)(x+3)} - \dfrac{2x}{(x+4)(x+4)}$ LCD: $(x+4)(x+4)(x+2)(x+3)$

$\dfrac{3x(x+4)(x+4) - 2x(x+2)(x+3)}{(x+4)(x+4)(x+2)(x+3)} = \dfrac{3x(x^2+8x+16) - 2x(x^2+5x+6)}{(x+4)(x+4)(x+2)(x+3)}$

$= \dfrac{3x^3+24x^2+48x-2x^3-10x^2-12x}{(x+4)^2(x+2)(x+3)} = \boxed{\dfrac{x^3+14x^2+36x}{(x+4)^2(x+2)(x+3)}}$

18. $\dfrac{2}{x^2-1} - 3$ LCD: (x^2-1) $\dfrac{2}{x^2-1} - \dfrac{3}{}\dfrac{(x^2-1)}{(x^2-1)} = \dfrac{2-3x^2+3}{x^2-1}$

$$\boxed{\dfrac{-3x^2+1}{x^2-1}}$$

19. $\dfrac{2x}{x-5} - \dfrac{x}{x+7}$ LCD: $(x-5)(x+7)$ $\left(\dfrac{x+7}{x+7}\right)\dfrac{2x}{x-5} - \dfrac{x}{x+7}\left(\dfrac{x+5}{x+5}\right)$

$\dfrac{2x(x+7) - x(x+5)}{(x+7)(x-5)} = \dfrac{2x^2+14x-x^2-5x}{(x+7)(x-5)} = \boxed{\dfrac{x^2+9x}{(x+7)(x-5)}}$

20. $\dfrac{x}{4} = \dfrac{x+1}{3}$

$3x = 4(x+1)$

$3x = 4x+4$

$-x = 4$

$x = -4$

21. $\dfrac{2}{x^2-1} = \dfrac{4}{x+1}$

$2(x+1) = 4(x^2-1)$

$2x+2 = 4x^2-4$

$4x^2-2x-6 = 0$

$2(2x^2-x-3) = 0$

$2x^2-x-3 = 0$

$x^2-x-6 = 0$

$(x-3)(x+2) = 0$

$\dfrac{(x-3)}{2}\dfrac{(x+2)}{2} = 0$

$(2x-3)(x+1) = 0$

$\boxed{x = 3/2}$ $x = -1 \leftarrow$ Does not check!

22. $\dfrac{3x}{5} + \dfrac{4}{x} = \dfrac{4x+1}{5}$ LCD: $5x$

$\left(\dfrac{x}{x}\right)\dfrac{3x}{5} + \dfrac{4}{x}\left(\dfrac{5}{5}\right) = \dfrac{4x+1}{5}\left(\dfrac{x}{x}\right)$

$\dfrac{3x^2}{5x} + \dfrac{20}{5x} = \dfrac{4x^2+x}{5x}$

$3x^2+20 = 4x^2+x$

$x^2+x-20 = 0$

$(x+5)(x-4) = 0$

$\boxed{\begin{array}{l} x = -5 \\ x = 4 \end{array}}$

23. $\dfrac{3x}{x-2} = 4 + \dfrac{x}{5}$ $\underline{LCD: 5(x-2)}$

$\dfrac{(5)}{(5)} \dfrac{3x}{x-2} = 4\left(\dfrac{5(x-2)}{5(x-2)}\right) + \dfrac{x}{5}\dfrac{(x-2)}{(x-2)}$

$\dfrac{15x}{5(x-2)} = \dfrac{4(5x-10)}{5(x-2)} + \dfrac{x^2-2x}{5(x-2)}$

$15x = 20x - 40 + x^2 - 2x$

$15x = x^2 + 18x - 40$

$x^2 + 3x - 40 = 0$

$(x+8)(x-5) = 0$

$\boxed{x = -8 \quad x = 5}$

24. $x + \dfrac{x}{4} - \dfrac{x}{5} = 21$ $\underline{LCD: 20}$

$\left(\dfrac{20}{20}\right)x + \dfrac{x}{4}\left(\dfrac{5}{5}\right) - \dfrac{x}{5}\left(\dfrac{4}{4}\right) = 21\left(\dfrac{20}{20}\right)$

$\dfrac{20x}{20} + \dfrac{5x}{20} - \dfrac{4x}{20} = \dfrac{420}{20}$

$21x = 420$

$\boxed{x = 20}$

25. $\dfrac{3}{x+4} + \dfrac{5}{4} = \dfrac{18}{x+4}$ $\underline{LCD: 4(x+4)}$

$\left(\dfrac{4}{4}\right)\dfrac{3}{x+4} + \dfrac{5}{4}\left(\dfrac{x+4}{x+4}\right) = \dfrac{18}{x+4}\left(\dfrac{4}{4}\right)$ $\quad 12 + 5x + 20 = 72$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad 5x + 32 = 72 \quad \boxed{x = 8}$

$\dfrac{12}{4(x+4)} + \dfrac{5x+20}{4(x+4)} = \dfrac{72}{4(x+4)}$ $\qquad 5x = 40$

26.

$$\frac{4}{x} + \frac{4}{x+1.5} = 1 \qquad \text{LCD: } x(x+1.5)$$

$$\frac{(x+1.5)}{(x+1.5)} \frac{4}{x} + \frac{4}{x+1.5}\left(\frac{x}{x}\right) = \frac{x(x+1.5)}{x(x+1.5)}$$

$$\frac{4x+6}{x(x+1.5)} + \frac{4x}{x(x+1.5)} = \frac{x^2+1.5x}{x(x+1.5)}$$

$$8x+6 = x^2+1.5x \qquad\qquad x = -0.8 \leftarrow \text{Time can't be negative!}$$

$$x^2 - 6.5x - 6 = 0 \qquad\qquad x = 7.3$$

graph, find intersections $\qquad 7.3 + 1.5 = \boxed{8.8 \text{ hours}}$

27.

$$\frac{7.75}{x} + \frac{7.75}{x-1} = 1 \qquad \text{LCD: } (x)(x-1)$$

$$\left(\frac{x-1}{x-1}\right)\frac{7.75}{x} + \frac{7.75}{x-1}\left(\frac{x}{x}\right) = \frac{(x)(x-1)}{(x)(x-1)}$$

$$\frac{7.75x - 7.75}{x(x-1)} + \frac{7.75x}{x(x-1)} = \frac{x^2-x}{(x)(x-1)}$$

$$15.5x - 7.75 = x^2 - x$$

$$-7.75 = x^2 - 16.5x \qquad\qquad x = 0.48 \leftarrow \text{does not make sense in context of problem}$$

$$x^2 - 16.5x + 7.75 = 0 \qquad\qquad \boxed{x = 16.02}$$

graph, find intersections