# Unit 2 Study Guide Solutions

1a. $\sqrt{-16} = i\sqrt{16} = \boxed{\pm 4i}$

b. $\sqrt{-8} \cdot \sqrt{-12} = i\sqrt{8} \cdot i\sqrt{12} = 2i\sqrt{2} \cdot 2i\sqrt{3} = 4i^2\sqrt{6} = \boxed{-4\sqrt{6}}$

c. $(6i)(-2i) = -12i^2 = \boxed{12}$

d. $i^{30} = (i^2)^{15} = (-1)^{15} = \boxed{-1}$

e. $i^{15} = i \cdot i^{14} = i \cdot (i^2)^7 = i(-1)^7 = \boxed{-i}$

f. $(5-2i)-(8-11i) = 5-2i-8+11i = \boxed{-3+9i}$

g. $(13i-2)5i = 65i^2 - 10i = \boxed{-65-10i}$

h. $(3+4i)(5-2i) = 15-6i+20i-8i^2 = 15+14i+8 = \boxed{23+14i}$

i. $(14-5i)^2 = (14-5i)(14-5i) = 196-70i-70i+25i^2$

$= 196-140i-25$

$= \boxed{171-140i}$

2a. $(5+3i)(5-3i) = 25-15i+15i-9i^2$

$= 25+9$

$= \boxed{34}$

b. $(3-4i)(3+4i) = 9+12i-12i-16i^2$

$= 9+16$

$= \boxed{25}$

3a. $\dfrac{3}{(6-2i)} \cdot \dfrac{(6+2i)}{(6+2i)} = \dfrac{18+6i}{36+12i-12i-4i^2} = \dfrac{18+6i}{36+4} = \dfrac{18+6i}{40} = \boxed{\dfrac{9+3i}{20}}$

*Fully simplified! (aka reduced!)*

b. $\dfrac{(3-7i)}{(5-4i)} \cdot \dfrac{(5+4i)}{(5+4i)} = \dfrac{15+12i-35i-28i^2}{25-20i+20i-16i^2} = \dfrac{15-23i+28}{25+16} = \boxed{\dfrac{43-23i}{41}}$

4. $\sqrt{64x^5y^8} = 8\sqrt{\overline{xxxx}\,x}$      $= \boxed{8x^2y^4\sqrt{x}}$
$\phantom{4.}\underline{(yyyyyyyy)}$

5. $\sqrt{80} + \sqrt{45} - \sqrt{72} = \sqrt{(2\cdot2)(2\cdot2)5} + \sqrt{(3\cdot3)5} - \sqrt{(2\cdot2)2(3\cdot3)}$

$\begin{array}{ccc} 8\wedge10 & 9\wedge5 & 9\wedge8 \\ 4\wedge2\ 2\wedge5 & 3\ 3 & 3\ 3\ 4\wedge2 \\ 2\ 2 & & 2\ 2 \end{array}$

$\phantom{5.}\qquad\qquad\qquad\qquad = \quad 4\sqrt{5} \quad + \quad 3\sqrt{5} \quad - \quad 6\sqrt{2}$

$\phantom{5.}\qquad\qquad\qquad\qquad = \qquad \boxed{7\sqrt{5} - 6\sqrt{2}}$

6. $\sqrt[3]{6x^2} \cdot \sqrt[3]{4x^5} = \sqrt[3]{24x^7} = \sqrt[3]{(2\cdot2\cdot2)3}\ \overline{xxx}\,\overline{xxx}\,x = \boxed{2x^2\sqrt[3]{3x}}$

$\phantom{6.}\qquad\qquad\qquad\qquad \begin{array}{c} 6\wedge\ 4\wedge \\ 3\ 2\ 2\ 2 \end{array}$

7. $\dfrac{5}{\sqrt{125}} \cdot \dfrac{\sqrt{125}}{\sqrt{125}} = \dfrac{5\sqrt{125}}{125} = \dfrac{\sqrt{125}}{25} = \dfrac{\sqrt{125}}{25}\begin{smallmatrix}5\\25\wedge\\5\,5\end{smallmatrix} = \dfrac{5\sqrt{5}}{25} = \boxed{\dfrac{\sqrt{5}}{5}}$

8. $2 + \sqrt{x+9} = 5$      check: $2 + \sqrt{0+9} = 5$

$\sqrt{x+9}^2 = (3)^2$           $2 + \sqrt{9} = 5$

$x + 9 = 9$              $2 + 3 = 5$      $\boxed{x = 0}$

$x = 0$                 $5 = 5\checkmark$

9. $(4x-2)^{\frac{1}{3}} + 2 = 620$     check: $(4(59007258.5)-2)^{\frac{1}{3}} + 2 = 620$

$(4x-2)^{\frac{1}{3}\cdot\frac{3}{1}} = (618)^{\frac{3}{1}}$          $(236029032)^{\frac{1}{3}} + 2 = 620$

$4x - 2 = 236029032$               $618 + 2 = 620$

$4x = 236029034$                 $620 = 620\checkmark$

$\boxed{x = 59007258.5}$

10. $(2 + \sqrt{x-6})^2 = (\sqrt{x+10})^2$

check:

$(2+\sqrt{x-6})(2+\sqrt{x-6}) = x + 10$ 　　　$2 + \sqrt{15-6} = \sqrt{15+10}$

$4 + 4\sqrt{x-6} + x - 6 = x + 10$ 　　　$2 + \sqrt{9} = \sqrt{25}$

$4\sqrt{x-6} + x - 2 = x + 10$ 　　　$2 + 3 = 5$

$4\sqrt{x-6} = 12$ 　　　$5 = 5 \checkmark$

$\sqrt{x-6}^2 = (3)^2$

$x - 6 = 9$

$\boxed{x = 15}$

11. $3 + (4-x)^{\frac{3}{2}} = 11$ 　　　check: $3 + (4-0)^{\frac{3}{2}} = 11$

$(4-x)^{3/2 \cdot \frac{2}{3}} = (8)^{2/3}$ 　　　$3 + (4)^{3/2} = 11$

$4 - x = 4$ 　　　$3 + 8 = 11$

$-x = 0$ 　　　$11 = 11 \checkmark$

$\boxed{x = 0}$

12. $5 + \sqrt{x+7} = x$ 　　　check: $5 + \sqrt{9+7} = 9$

$\sqrt{x+7} = x - 5$ 　　　$5 + \sqrt{16} = 9$

$x + 7 = (x-5)(x-5)$ 　　　$5 + 4 = 9$

$x + 7 = x^2 - 10x + 25$ 　　　$9 = 9 \checkmark$

$0 = x^2 - 11x + 18$

18

9 ⤬ -2

-11

$(x-9)(x-2) = 0$ 　　　$5 + \sqrt{2+7} = 2$

$x - 9 = 0 \quad x - 2 = 0$ 　　　$5 + \sqrt{9} = 2$

$x = 9 \qquad x = 2 \Leftarrow$ extraneous! 　　　$5 + 3 = 2$

　　　$8 \neq 2$

13. $\left(\dfrac{x^4 \, y^{-2}}{x^{-3} \, y^5}\right)^{-1} = \dfrac{x^{-4} \, y^2}{x^3 \, y^{-5}} = \dfrac{y^2 \, y^5}{x^3 \, x^4} = \boxed{\dfrac{y^7}{x^7}}$

14. $\left(\dfrac{9a^{-3}}{18b^{-4}}\right)^2 = \dfrac{81a^{-6}}{324b^{-8}} = \dfrac{81b^8}{324a^6} = \boxed{\dfrac{b^8}{4a^6}}$

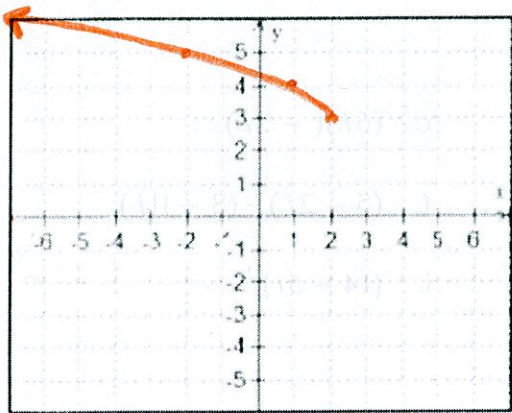15. $\sqrt[3]{(27p^6)^5} = \boxed{\sqrt[3]{14348907\,p^{30}}}$

16. $\sqrt[4]{m^3} = \boxed{m^{3/4}}$

17. see study guide

18. see study guide

**Goal 5:** Graphing Radical Functions

17. Graph $y = \sqrt{-x+2} + 3$

| X | Y |
|---|---|
| -2 | 5 |
| 1 | 4 |
| 2 | 3 |
| 3 | ERROR |

$-x + 2 = 0 \quad x = 2$
$-x = -2$ ↗

Translations:
right 2, up 3, reflect over y-axis
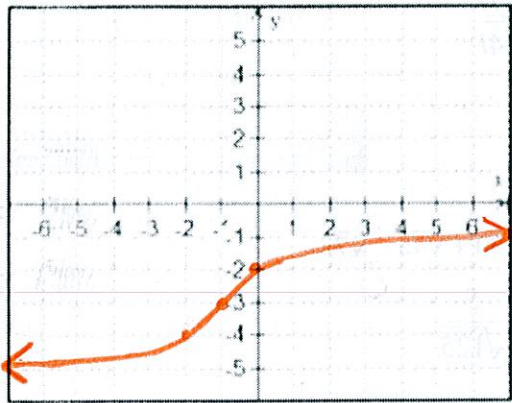
Key Point:
$(2, 3)$

Domain:
$(-\infty, -2]$

Range:
$[3, \infty)$

End Behavior:
$y \to 3, \quad x \to 2$
$y \to \infty, \quad x \to -\infty$

18. Graph $y = \sqrt[3]{x+1} - 3$

| X | Y |
|---|---|
| -2 | -4 |
| -1 | -3 |
| 0 | -2 |
| 7 | -1 |

Translations:
Left 1, Down 3

Domain:
$(-\infty, \infty)$

Range:
$(-\infty, \infty)$

End Behavior:
$y \to -\infty, \quad x \to -\infty$
$y \to \infty, \quad x \to \infty$

# Unit 2: Radicals and Complex Numbers

Math 3 Study Guide

Name: _____

## Goal 1: Simplifying Imaginary Numbers

1. Simplify each of the following:

   a. $\sqrt{-16}$

   b. $\sqrt{-8} \cdot \sqrt{-12}$

   c. $(6i)(-2i)$

   d. $i^{30}$

   e. $i^{15}$

   f. $(5-2i)-(8-11i)$

   g. $(13i-2)5i$

   h. $(3+4i)(5-2i)$

   i. $(14-5i)^2$

2. Multiply given expression to its conjugate and give the resulting value

   a. $5+3i$

   b. $3-4i$

3. Simplify (no $i$'s in denominator)

   a. $\dfrac{3}{6-2i}$

   b. $\dfrac{3-7i}{5-4i}$

## Goal 2: Simplifying Radical Expressions

4. Simplify $\sqrt{64x^5y^8}$

5. Simplify $\sqrt{80} + \sqrt{45} - \sqrt{72}$

6. Multiply $\sqrt[3]{6x^2} \cdot \sqrt[3]{4x^5}$

7. Simplify: $\dfrac{5}{\sqrt{125}}$

## Goal 3: Solving Radical Equations

8. Solve for x: $2 + \sqrt{x+9} = 5$

9. Solve: $(4x-2)^{\frac{1}{3}} + 2 = 620$

10. Solve for x: $2 + \sqrt{x-6} = \sqrt{x+10}$?

11. Solve for x: $3 + (4-x)^{\frac{3}{2}} = 11$

12. What is the extraneous solution to the equation $5 + \sqrt{x+7} = x$?

## Goal 4: Simplifying Expressions with Exponents

13. Simplify $\left(\dfrac{x^4 y^{-2}}{x^{-3} y^5}\right)^{-1}$

14. Simplify $\left(\dfrac{9a^{-3}}{18b^{-4}}\right)^2$

15. Write as a radical: $(27p^6)^{\frac{5}{3}}$

16. Write using rational exponents: $\left(\sqrt[4]{m}\right)^3$