

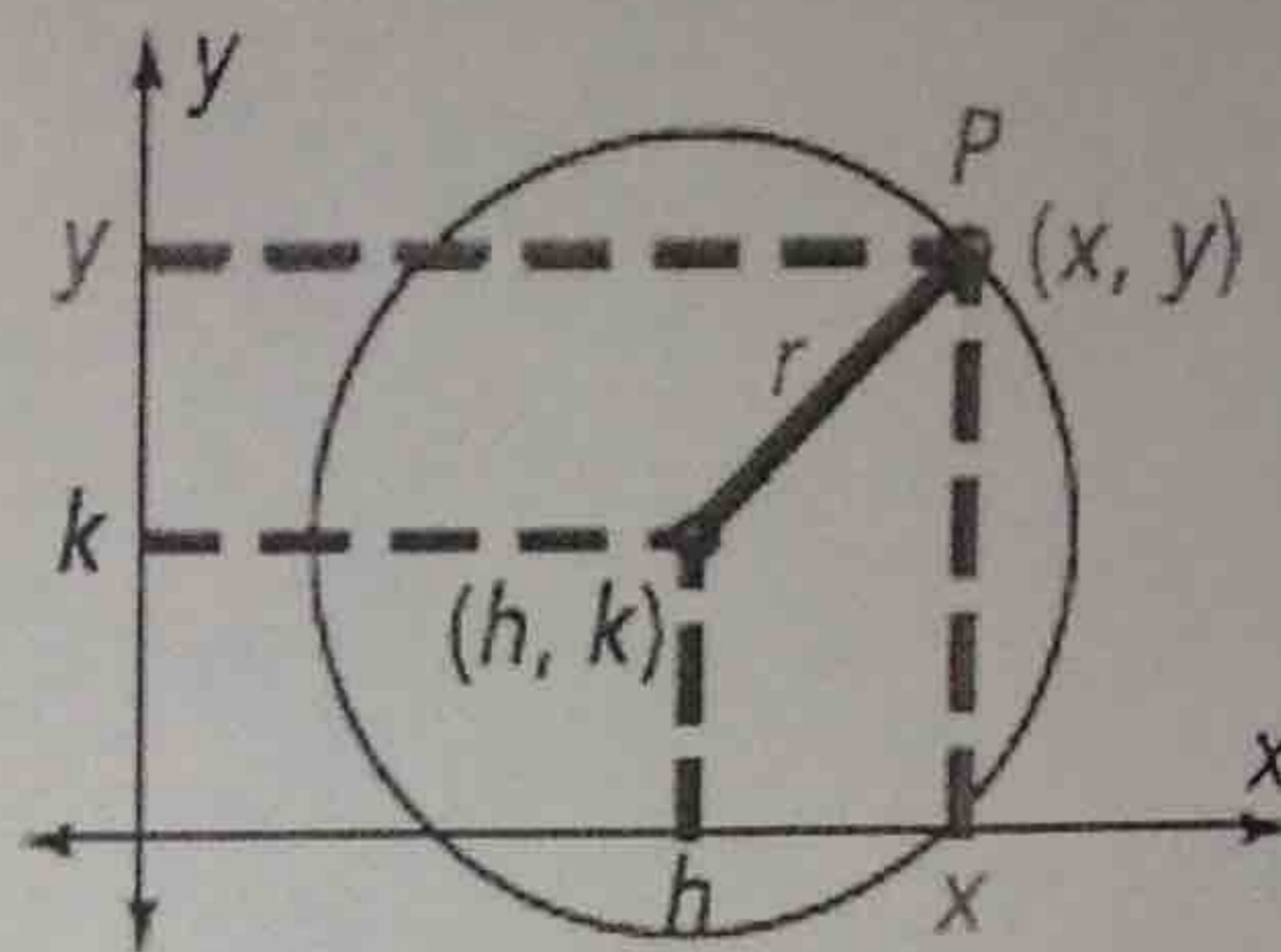
8.8 Equations of Circles

SWBAT graph circles on the coordinate plane and write the equations of circles in standard form.

Standard Form of Circles

$$(x-h)^2 + (y-k)^2 = r^2$$

Center:	Radius:	Point on the circle:
(h, k)	r	(x, y)



Example 1: Write the equation of a circle with the given information.

a) Center (0,0), Radius=10

$h = 0 \quad k = 0 \quad r = 10$

$$x^2 + y^2 = 100$$

b) Center (2, 3), Diameter=12

$h = 2 \quad k = 3 \quad r = 6$

$$(x-2)^2 + (y-3)^2 = 36$$

Example 2: Determine the center and radius of a circle the given equation.

a) $x^2 + y^2 = \frac{9}{4}$

$(0, 0) \quad r = 3/2$

b) $(x+3)^2 + (y-5)^2 = 81$

$(-3, 5) \quad r = 9$

c) $(x+4)^2 + (y+6)^2 = 1$

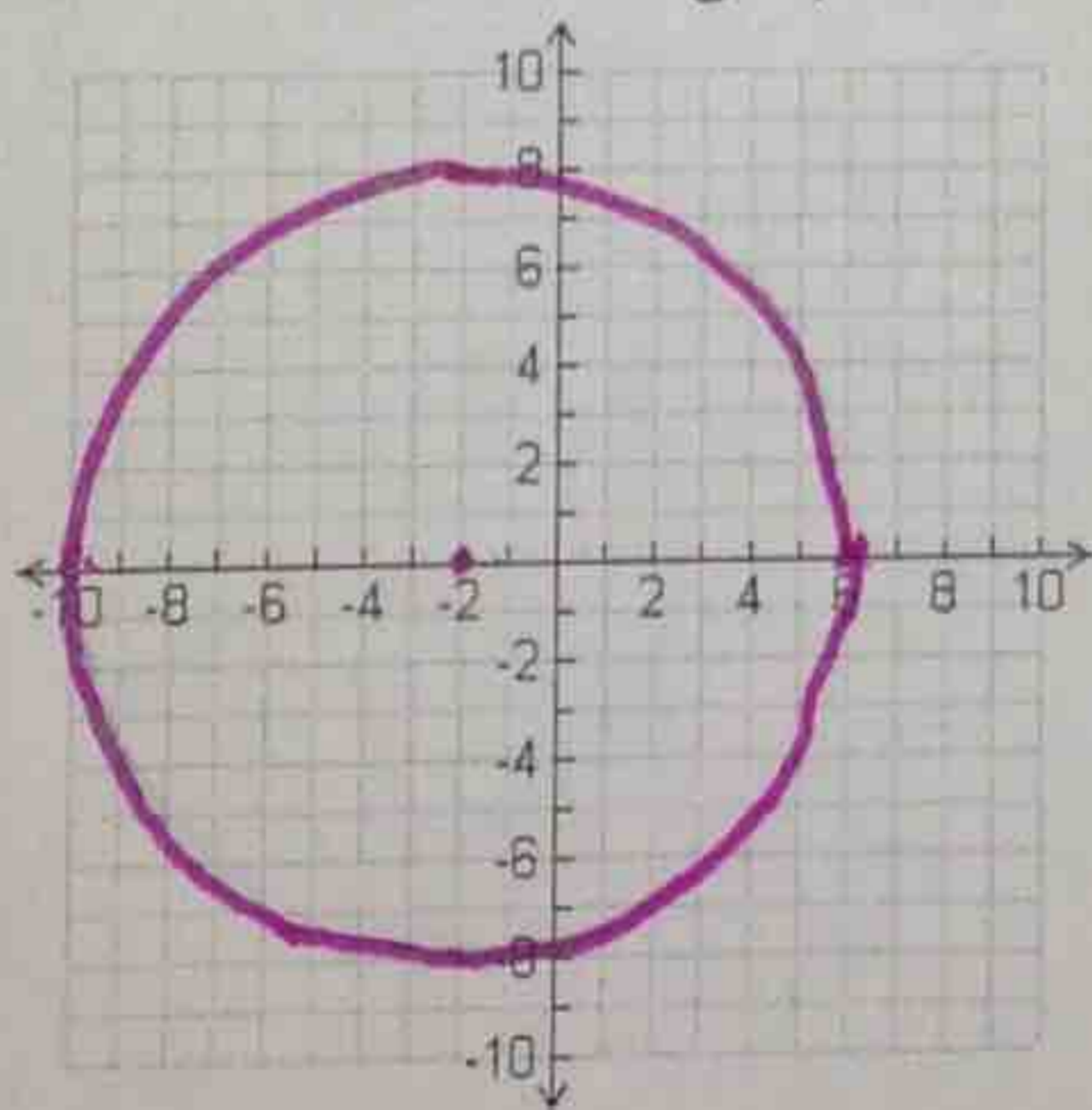
$(-4, -6) \quad r = 1$

Example 3: Use the center and the radius to graph each circle.

a) $(x+2)^2 + y^2 = 64$

Center: $(-2, 0)$

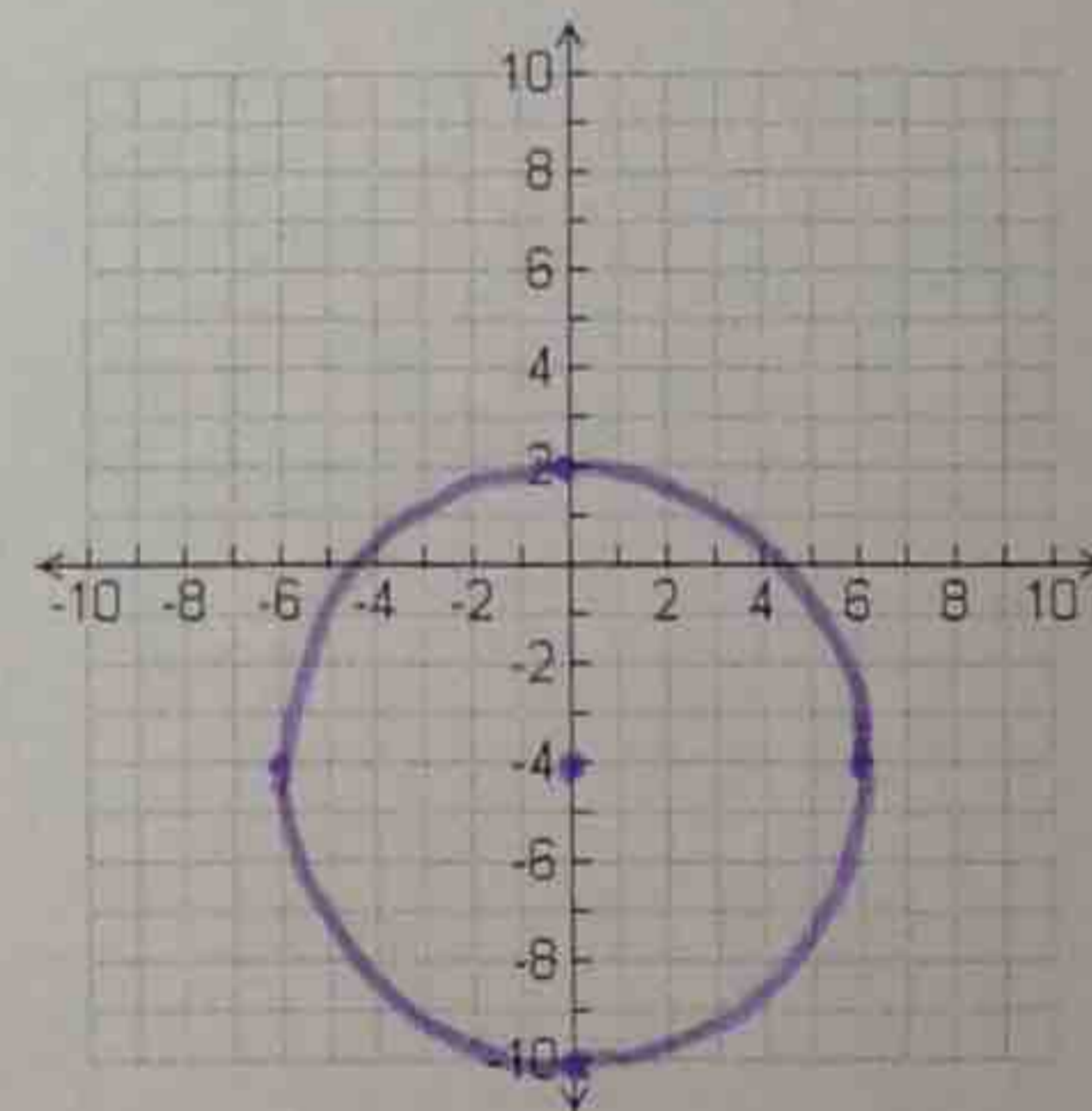
Radius: 8



b) $x^2 + (y+4)^2 = 36$

Center: $(0, -4)$

Radius: 6



Writing an Equation with a Pass-Thru Point

Step 1: Substitute the center (h, k) into the equation

Step 2: Substitute the "pass through point (x, y) " into the equation for x and y .

Step 3: Simplify and solve for r^2 .

Step 4: Substitute r^2 back into the equation from Step 1.

Example 4: Write the equation of a circle with a given center $(2, 5)$ that passes through the point $(5, -1)$.

$$(x-2)^2 + (y-5)^2 = r^2$$

$$(5-2)^2 + (-1-5)^2 = r^2$$

$$9 + 36 = r^2$$

$$r^2 = 45$$

$$(x-2)^2 + (y-5)^2 = 45$$

(x_c, y_c)

Writing an Equation with Two Points on the Circle

Midpoint Formula

Find the midpoint (center) between the two endpoints, and then follow steps 1-4.

$$x_c = \frac{x_2 + x_1}{2} \quad y_c = \frac{y_2 + y_1}{2}$$

Example 5: Write the equation of a circle with endpoints of diameter at $(-6, 5)$ and $(4, -3)$.

$$x_c = \frac{-6 + 4}{2} = -1$$

$$y_c = \frac{5 - 3}{2} = 1$$

$$\begin{aligned} (x+1)^2 + (y-1)^2 &= r^2 \\ (-6+1)^2 + (5-1)^2 &= r^2 \\ 25 + 16 &= r^2 \\ 41 &= r^2 \end{aligned}$$

$$(x+1)^2 + (y-1)^2 = 41$$

$(-1, 1) = \text{center}$

Writing the Equation of a Circle in Standard Form

Step 1:	Group x's and group y's together.
Step 2:	Move any constants to the right side of the equation.
Step 3:	Use complete the square to make a perfect square trinomial for the x's and then again for the y's. <i>*Remember, whatever you do to one side of the equation, you must do to the other!</i>
Step 4:	Simplify factors into standard form of a circle!

Example 5: Write the equation of a circle in standard form. Then, state the center and the radius.

a) $x^2 + y^2 + 4x - 8y + 16 = 0$

b) $x^2 + y^2 + 6x - 4y = 0$

$$\begin{aligned} x^2 + 4x + y^2 - 8y &= -16 \\ (x^2 + 4x + 4) + (y^2 - 8y + 16) &= -16 + 4 + 16 \\ (x+2)^2 + (y-4)^2 &= 4 \end{aligned}$$

center = $(-2, 4)$
 $r = 2$

$$\begin{aligned} (x^2 + 6x + 9) + (y^2 - 4y + 4) &= 9 + 4 \\ (x+3)^2 + (y-2)^2 &= 13 \end{aligned}$$

center = $(-3, 2)$
 $r = \sqrt{13}$

c) $x^2 + y^2 - 6x - 2y + 4 = 0$

d) $x^2 + y^2 + 8x - 10y - 4 = 0$

$$\begin{aligned} (x^2 - 6x + 9) + (y^2 - 2y + 1) &= -4 + 9 + 1 \\ (x-3)^2 + (y-1)^2 &= 6 \end{aligned}$$

center = $(3, 1)$
 $r = \sqrt{6}$

$$\begin{aligned} (x^2 + 8x + 16) + (y^2 - 10y + 25) &= 4 + 16 + 25 \\ (x+4)^2 + (y-5)^2 &= 45 \end{aligned}$$

center = $(-4, 5)$
 $r = \sqrt{45} = 3\sqrt{5}$