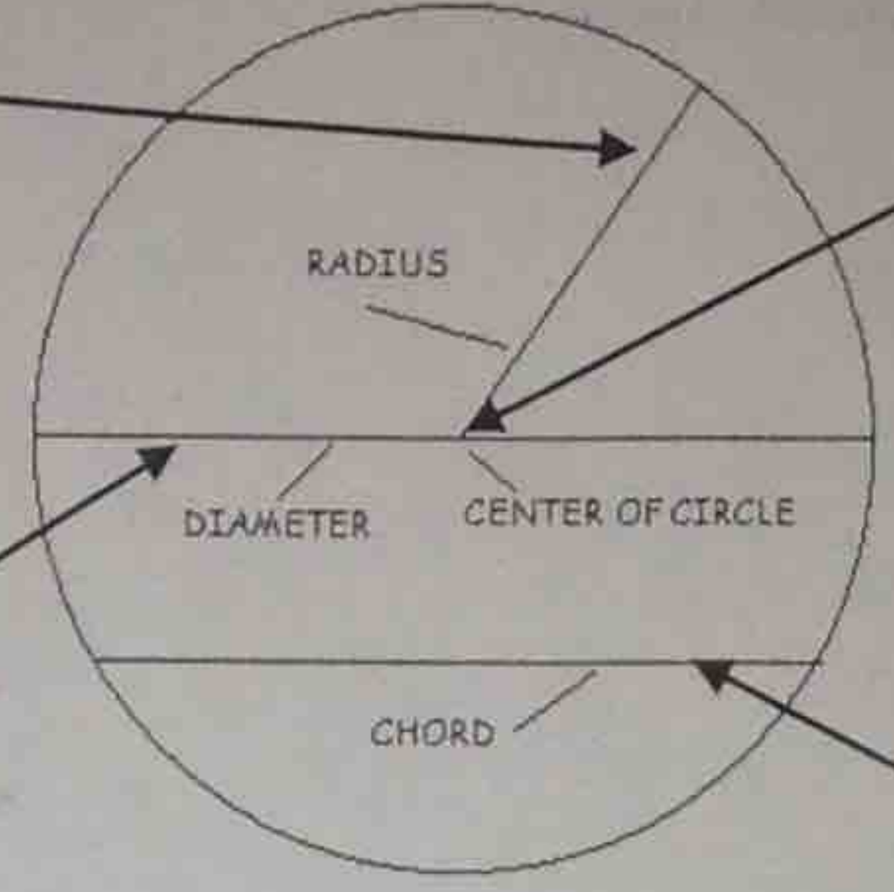
8.2 Chords & Arcs of Circles

SWBAT solve for unknown variables using theorems about chords and arcs of circles.

Any segment with endpoints that are the center and a point on the circle is a radius.

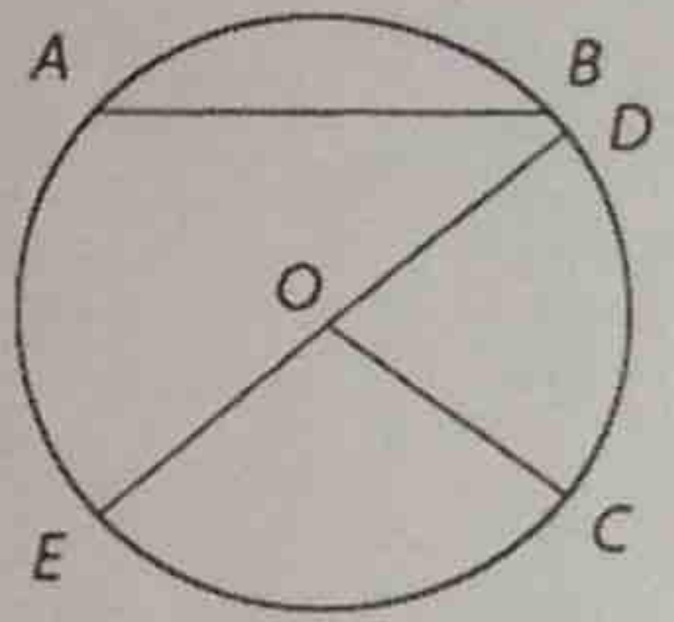
A segment that passes through the center is a diameter of a circle.



The given point is called the center. This point names the circle.

Any segment with endpoints that are on a circle is called a chord.

Example 1: Name the circle, a radius, a chord, and a diameter of the circle.

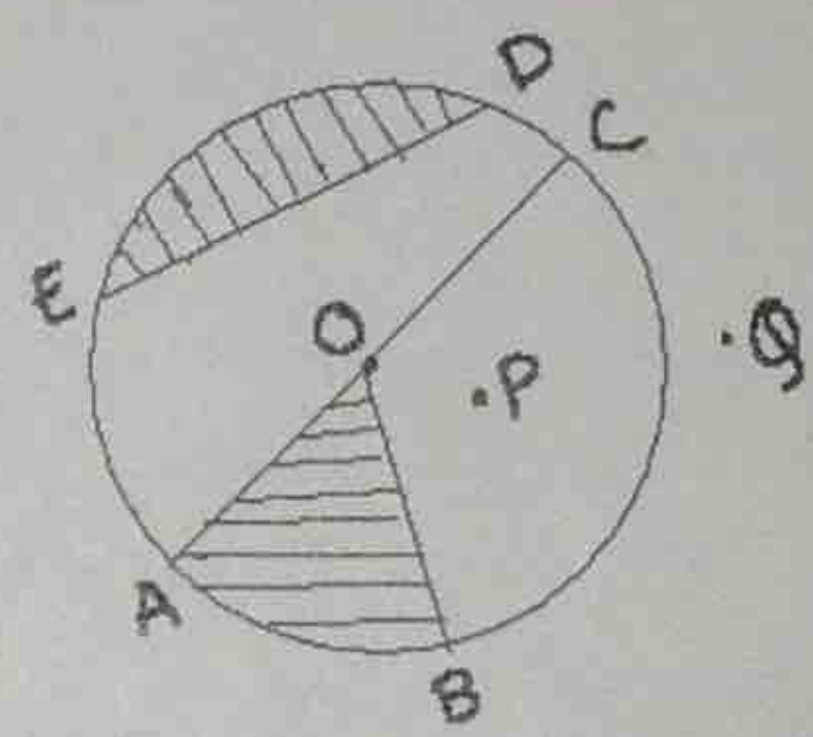


Circle: ⊙O

Radius: $\overline{OD}$

Chord: $\overline{AB}$

Diameter: $\overline{DE}$



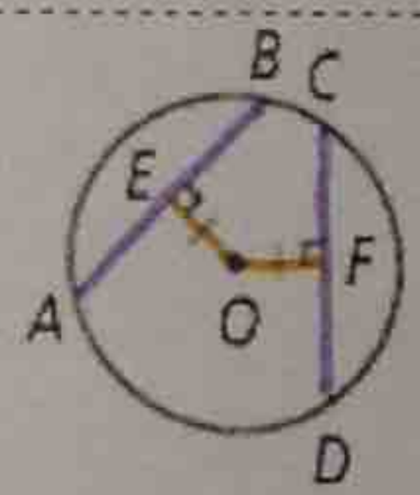
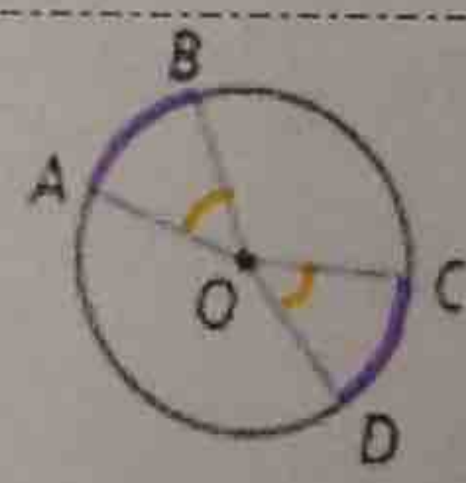
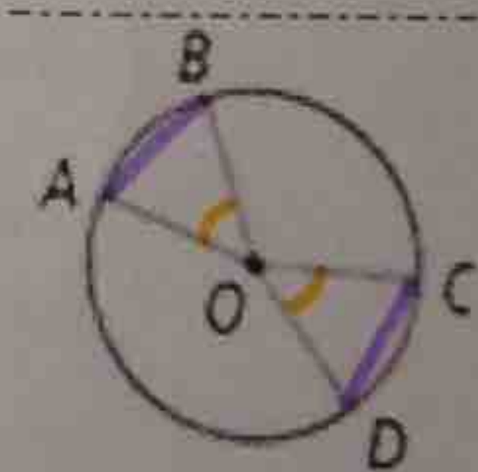
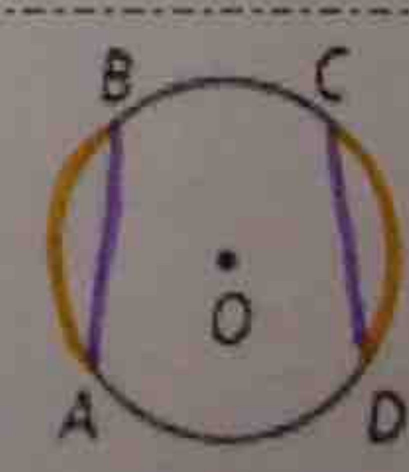
Circle: ⊙O

Radius: $\overline{OB}$

Chord: $\overline{ED}$

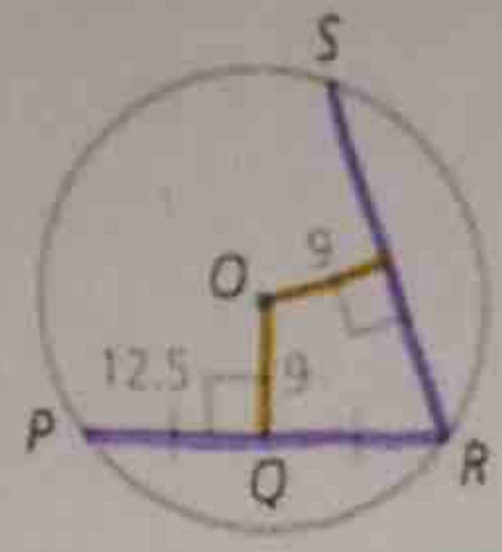
Diameter: $\overline{AC}$

Since a diameter is composed of two radii, then **d = 2r** and **r = d/2**

Theorem 1:	Converse Theorem 1:	
Within a circle or in congruent circles, chords equidistant from the center or centers are congruent. If $OE = OF$, then $\overline{AB} \cong \overline{CD}$.	Within a circle or in congruent circles, congruent chords are equidistant from the center (or centers). If $\overline{AB} \cong \overline{CD}$, then $OE = OF$.	
Theorem 2:	**Converse Theorem 2:**	
Within a circle or in congruent circles, congruent central angles have congruent arcs. If $\angle AOB \cong \angle COD$, then $\overset{\frown}{AB} \cong \overset{\frown}{CD}$.	Within a circle or in congruent circles, congruent arcs have congruent central angles. If $\overset{\frown}{AB} \cong \overset{\frown}{CD}$, then $\angle AOB \cong \angle COD$.	
Theorem 3:	**Converse Theorem 3:**	
Within a circle or in congruent circles, congruent central angles have congruent chords. If $\angle AOB \cong \angle COD$, then $\overline{AB} \cong \overline{CD}$.	Within a circle or in congruent circles, congruent chords have congruent central angles. If $\overline{AB} \cong \overline{CD}$, then $\angle AOB \cong \angle COD$.	
Theorem 4:	**Converse Theorem 4:**	
Within a circle or in congruent circles, congruent chords have congruent arcs. If $\overline{AB} \cong \overline{CD}$, then $\overset{\frown}{AB} \cong \overset{\frown}{CD}$.	Within a circle or in congruent circles, congruent arcs have congruent chords. If $\overset{\frown}{AB} \cong \overset{\frown}{CD}$, then $\overline{AB} \cong \overline{CD}$.	

Example 2: The following chords are equidistant from the center of the circle.

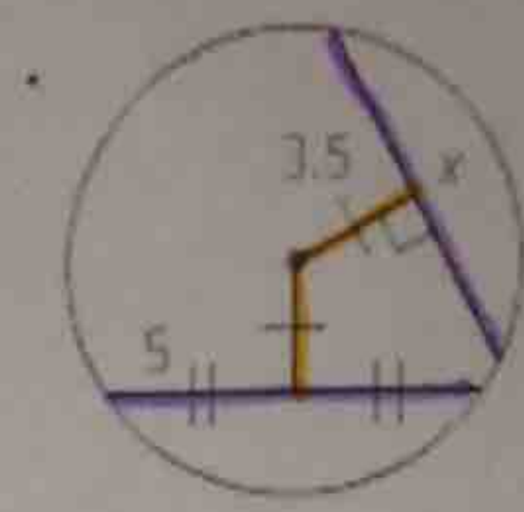
a) What is the length of RS?



$12.5 \times 2 = 25$

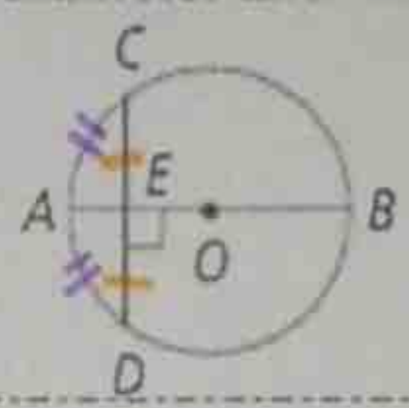
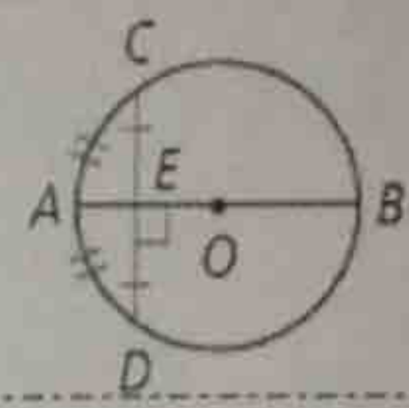
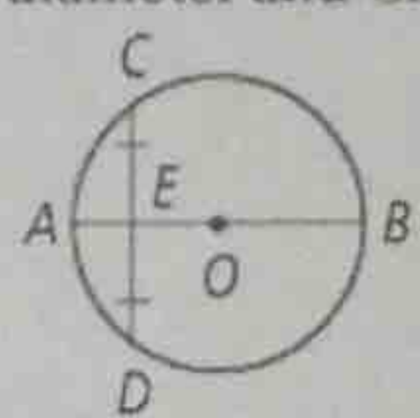
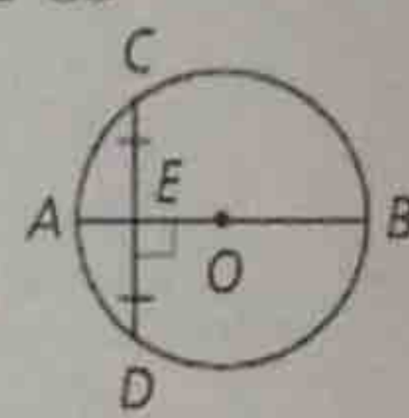
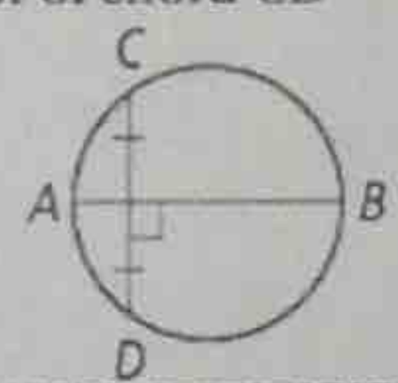
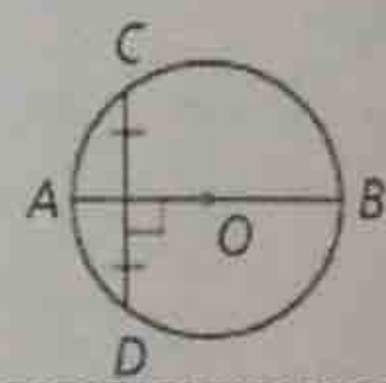
$\overline{RS} = 25$

b) Solve for x.

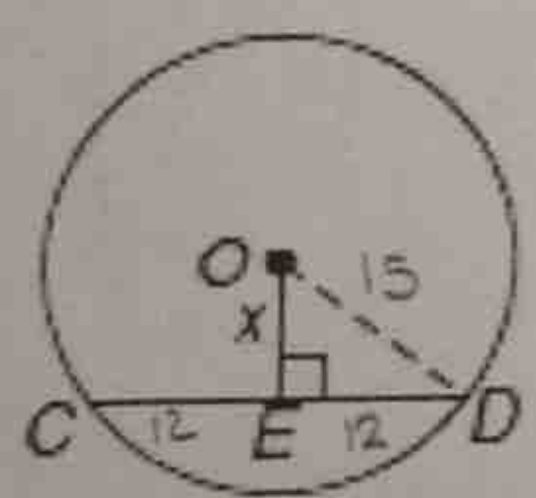


$5 \times 2 = 10$

$x = 10$

Theorem	If . . .	Then . . .
Theorem 5: In a circle, if a diameter is perpendicular to a chord, then it bisects the chord and its arc.	$\overline{AB}$ is a diameter and $\overline{AB} \perp \overline{CD}$ 	$\overline{CE} \cong \overline{ED}$ and $\widehat{CA} \cong \widehat{AD}$ 
Theorem 6: In a circle, if a diameter bisects a chord that is not a diameter, then it is perpendicular to the chord.	$\overline{AB}$ is a diameter and $\overline{CE} \cong \overline{ED}$ 	$\overline{AB} \perp \overline{CD}$ 
Theorem 7: In a circle, the perpendicular bisector of a chord contains the center of the circle.	$\overline{AB}$ is the perpendicular bisector of chord $\overline{CD}$ 	$\overline{AB}$ contains the center of $\odot O$ 

Example 3: In $\odot O$, $\overline{CD} \perp \overline{OE}, OD = 15, and\ CD = 24$. Find x.

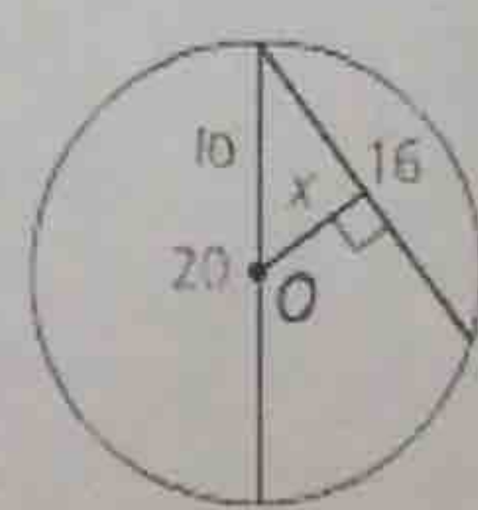


$x^2 + 12^2 = 15^2$

$x^2 = 81$

$x = 9$

Example 4: Find the value of x to the nearest tenth.



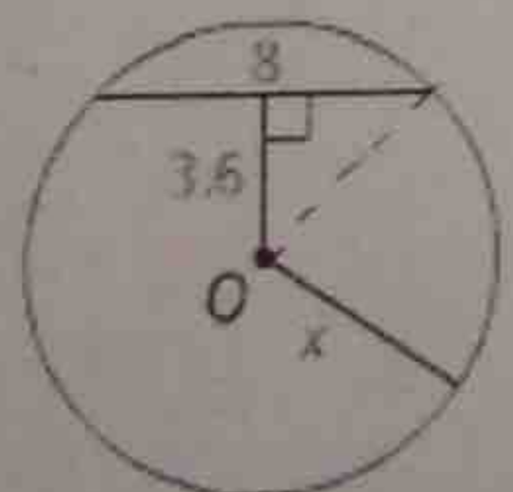
$x^2 + 8^2 = 10^2$

$x^2 = 36$

$x = 6$

You Try! Find the value of x to the nearest tenth.

a)

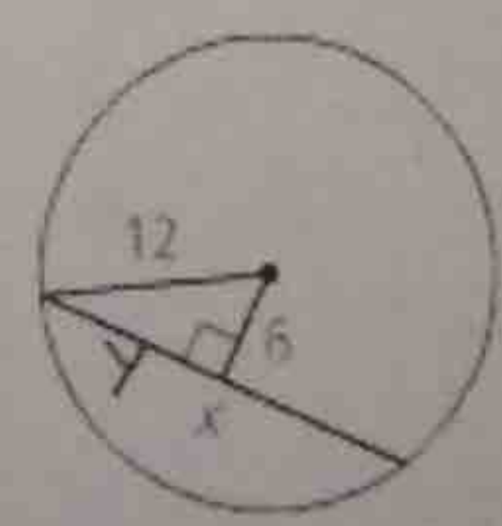


$3.6^2 + 4^2 = x^2$

$28.96 = x^2$

$x = 5.4$

b)



$y^2 + 6^2 = 12^2$

$y^2 = 108$

$y = 10.4$

$x = 2(10.4)$

$x = 20.8$