

5.4 Solving Rational Equations

SWBAT solve rational equations and real-life application problems.

How to Solve Rational Equations

1. Find a common denominators on each side of the equation
2. Simplify each side into one term, and then cross multiply.
3. Solve for the variable.
4. Check your answers for extraneous solutions!

Example 1: Solve $\frac{y}{5} + \frac{y}{2} = 7$

$$\frac{2y + 5y}{10} = 7$$

$$\frac{7y}{10} = 7$$

$$7y = 70$$

$$y = 10$$

You Try! Solve $\frac{3x-2}{12} - \frac{1}{6} = \frac{1}{6}$

$$\frac{3x-2-2}{12} = \frac{1}{6}$$

$$\frac{3x-4}{12} = \frac{1}{6}$$

$$6(3x-4) = 12$$

$$18x - 24 = 12$$

$$18x = 36$$

$$x = 2$$

Example 2: Solve $\frac{11}{3x} - \frac{1}{3} = \frac{-4}{x^2}$

$$\frac{11-x}{3x} = \frac{-4}{x^2}$$

$$x^2(11-x) = -4(3x)$$

$$11x^2 - x^3 = -12x$$

$$x^3 - 11x^2 - 12x = 0$$

$$x(x^2 - 11x - 12) = 0$$

$$x(x-12)(x+1) = 0$$

$$x = 0 \quad x = 12 \quad x = -1$$

↑ restriction

You Try! Solve $\frac{7}{4x} - \frac{3}{x^2} = \frac{1}{2x^2}$

$$\frac{7x-12}{4x^2} = \frac{1}{2x^2}$$

$$2x^2(7x-12) = 4x^2$$

$$14x^3 - 24x^2 - 4x^2 = 0$$

$$14x^3 - 28x^2 = 0$$

$$14x^2(x-2) = 0$$

$$x = 0 \quad x = 2$$

↑ extraneous (restriction)

Example 3: Solve $\frac{4}{x+2} + 3 = \frac{3x}{x-3}$

$$\frac{4+3(x+2)}{x+2} = \frac{3x}{x-3}$$

$$\frac{4+3x+6}{x+2} = \frac{3x}{x-3}$$

$$(x-3)(3x+10) = 3x(x+2)$$

$$3x^2 + 10x - 9x - 30 = 3x^2 + 6x$$

$$-5x - 30 = 0$$

$$-5x = 30$$

$$x = -6$$

You Try! Solve $\frac{5}{x-4} + 2 = \frac{2x}{x-3}$

$$\frac{5+2(x-4)}{x-4} = \frac{2x}{x-3}$$

$$\frac{5+2x-8}{x-4} = \frac{2x}{x-3}$$

$$(2x-3)(x-3) = 2x(x-4)$$

$$2x^2 - 6x - 3x + 9 = 2x^2 - 8x$$

$$-9x + 9 = -8x$$

$$9 = x$$

Example 4: Solve $\frac{x}{x-2} - 7 = \frac{2}{x-2}$

$$\frac{x-7(x-2)}{x-2} = \frac{2}{x-2}$$

$$x-7x+14 = 2$$

$$-6x+14 = 2$$

$$-6x = -12$$

$$x = 2$$

NO SOLUTION

If denominators are the same on both sides of the equation, cross them off & solve the numerator.

You Try! Solve $\frac{x-3}{x-4} = 4 + \frac{1}{x-4}$

$$\frac{x-3}{x-4} = \frac{4(x-4)+1}{x-4}$$

$$x-3 = 4x-16+1$$

$$x-3 = 4x-15$$

$$-3x-3 = -15$$

$$-3x = -12$$

$$x = 4$$

NO SOLUTION

Example 5: Solve $\frac{3}{x-3} - \frac{7}{x+3} = \frac{2}{x^2-9}$

$$\frac{3(x+3) - 7(x-3)}{(x+3)(x-3)} = \frac{2}{(x+3)(x-3)}$$

$$\begin{aligned} 3x+9-7x+21 &= 2 \\ -4x+30 &= 2 \\ -4x &= -28 \\ \boxed{x=7} \end{aligned}$$

You Try! Solve $\frac{4}{x-4} - \frac{3}{x+1} = \frac{5}{x^2-3x-4}$

$$\frac{4(x+1) - 3(x-4)}{(x-4)(x+1)} = \frac{5}{(x-4)(x+1)}$$

$$\begin{aligned} 4x+4-3x+12 &= 5 \\ x+16 &= 5 \\ \boxed{x=-11} \end{aligned}$$

Example 6: Solve $\frac{x}{x-4} = \frac{15}{x-3} - \frac{2x}{x^2-7x+12}$

$$\frac{x}{x-4} - \frac{15}{x-3} = \frac{-2x}{(x-4)(x-3)}$$

$$\frac{x(x-3) - 15(x-4)}{(x-4)(x-3)} = \frac{-2x}{(x-4)(x-3)}$$

$$\begin{aligned} x^2-3x-15x+60 &= -2x \quad (x-10)(x-6)=0 \\ x^2-18x+60 &= -2x \\ x^2-16x+60 &= 0 \\ \boxed{x=10 \quad x=6} \end{aligned}$$

You Try! Solve $\frac{3x}{x+2} - \frac{2}{x+3} = \frac{36}{x^2+5x+6}$

$$\frac{3x(x+3) - 2(x+2)}{(x+2)(x+3)} = \frac{36}{(x+2)(x+3)}$$

$$3x^2+9x-2x-4 = 36$$

$$3x^2+7x-4 = 36$$

$$3x^2+7x-40 = 0$$

$$(3x^2+15x-8x-40)$$

$$3x(x+5)-8(x+5)$$

$$\boxed{x=-5 \quad x=8/3}$$

Directions: Decide whether each of the following is an expression or an equation. If it is an equation, solve it. If it is an expression, write it as a single fraction.

1. $\frac{5x}{2y+4} - \frac{6}{y^2+2y}$ Expression

$$\frac{5xy}{2(y+2)y} + \frac{-6(2)}{y(y+2)(2)}$$

$$\boxed{\frac{5xy-12}{2y(y+2)}}$$

2. $\frac{1}{2x} - \frac{2}{5x} = \frac{1}{2}$ Equation

$$\frac{5-4}{10x} = \frac{1}{2}$$

$$\frac{1}{10x} = \frac{1}{2}$$

$$2 = 10x$$

$$\boxed{x = \frac{1}{5}}$$

3. $3x - \frac{x^2-5x}{x^2-2}$ Expression

$$\frac{3x(x^2-2) - (x^2-5x)}{x^2-2}$$

$$3x^3-6x-x^2+5x$$

$$\boxed{\frac{3x^3-x^2-x}{x^2-2}}$$

4. $\frac{-2}{x^2-2} = \frac{2}{x-4}$ Equation

$$-2(x-4) = 2(x^2-2)$$

$$-2x+8 = 2x^2-4$$

$$2x^2+2x-12 = 0$$

$$x^2+x-6 = 0$$

$$(x+3)(x-2) = 0$$

$$\boxed{x=-3 \quad x=2}$$

5. $\frac{7}{5y+25} - \frac{4}{3y+15}$ Expression

$$\frac{7(3)}{5(y+5)(3)} + \frac{-4(5)}{3(y+5)(5)}$$

$$\frac{21-20}{15(y+5)} = \boxed{\frac{1}{15(y+5)}}$$

$$x=0 \quad \boxed{x=4 \quad x=2} \rightarrow$$

↑
extraneous

6. $\frac{1}{x} - \frac{1}{6} = \frac{4}{3x^2}$ Equation

$$\frac{6-x}{6x} = \frac{4}{3x^2}$$

$$3x^2(6-x) = 4(6x)$$

$$18x^2-3x^3 = 24x$$

$$3x^3-18x^2+24x = 0$$

$$3x(x^2-6x+8) = 0$$

$$3x(x-4)(x-2) = 0$$