

5.2 Adding and Subtracting Rational Expressions

SWBAT simplify the addition and subtraction of rational expressions.

In order to add or subtract fractions, we must first find the Least Common Denominator

Arithmetic of Rational Numbers	Arithmetic of Rational Expressions
$a) \frac{4}{4} \frac{1}{3} + \frac{3}{4} \frac{3}{3} = \frac{4+9}{12} = \frac{13}{12}$	$a) \frac{(x-1)}{(x-1)} \frac{3}{(x+1)} + \frac{4}{(x-1)} \frac{(x+1)}{(x+1)} = \frac{3x-3+4x+4}{(x-1)(x+1)} = \frac{7x+1}{(x-1)(x+1)}$
$b) \frac{3}{3} \frac{9}{8} + \frac{5}{6} \frac{4}{4} = \frac{9+20}{24} = \frac{29}{24}$	$b) \frac{(x-1)}{(x-1)} \frac{2x}{(x+3)} + \frac{4x}{(x-1)} \frac{(x+3)}{(x+3)} = \frac{2x^2-2x+4x}{(x-1)(x+3)} = \frac{2x^2+2x}{(x-1)(x+3)}$
$c) \frac{3}{3} \frac{7}{8} - \frac{1}{6} \frac{4}{4} = \frac{21-4}{24} = \frac{17}{24}$	$c) \frac{(x-1)}{(x-1)} \frac{2x}{(x+3)} - \frac{4}{(x-1)} \frac{(x+3)}{(x+3)} = \frac{2x^2-2x-4x-12}{(x-1)(x+3)} = \frac{2x^2-6x-12}{(x-1)(x+3)}$

Monomial Denominators-FIND A COMMON DENOMINATOR!

- determine what each denominator has that the other denominator is missing
- multiply top and bottom by whatever is missing-to give you the common denominator

Example 1: $\frac{1}{6x} + \frac{2}{3x} - \frac{3}{4x}$ CD: $12x$

$$\frac{2(1)}{2(6x)} + \frac{4(2)}{4(3x)} + \frac{(-3)(3)}{(4x)3}$$

$$\frac{2+8-9}{12x} = \frac{1}{12x} \quad x \neq 0$$

Example 2: $\frac{5y+2}{xy^2} + \frac{2x-4}{4xy}$ CD: $4xy^2$

$$\frac{4(5y+2)}{4(xy^2)} + \frac{y(2x-4)}{y(4xy)} = \frac{20y+8+2xy-4y}{4xy^2}$$

$$\frac{2xy+16y+8}{4xy^2} = \frac{2(xy+8y+4)}{4xy^2} = \frac{xy+8y+4}{2xy^2}$$

$x \neq 0 \quad y \neq 0$

Example 3: $\frac{3}{7x^2y} + \frac{4}{21xy^2}$ CD: $21x^2y^2$

$$\frac{(3)3y}{(7x^2y)3y} + \frac{4x}{(21xy^2)x}$$

$$\frac{9y+4x}{21x^2y^2} \quad x \neq 0 \quad y \neq 0$$

Example 4: $\frac{3}{8x^3y^3} - \frac{1}{4xy}$ CD: $8x^3y^3$

$$\frac{3}{8x^3y^3} + \frac{-1(2x^2y)}{(4xy)(2x^2y)}$$

$$\frac{3-2x^2y^2}{8x^3y^3} \quad x \neq 0 \quad y \neq 0$$

Bi/Trinomial Denominators - FACTOR & FIND A COMMON DENOMINATOR!

- Always start by factoring polynomial denominators
- Multiply top and bottom by whatever is missing and then combine the numerators

Example 5: $\frac{w+12}{4w-16} - \frac{w+4}{2w-8}$ CD: $4(w-4)$

$$\frac{w+12}{4(w-4)} + \frac{(-w-4)2}{2(w-4)2} = \frac{w+12-2w-8}{4(w-4)}$$

$$\frac{-w+4}{4(w-4)} = \frac{-1(w-4)}{4(w-4)} = \boxed{\frac{-1}{4} \quad w \neq 4}$$

Example 6: $\frac{y}{2y+4} - \frac{3}{y+2}$ CD: $2(y+2)$

$$\frac{y}{2(y+2)} + \frac{-3(2)}{(y+2)2} = \boxed{\frac{y-6}{2(y+2)} \quad y \neq -2}$$

Example 7: $\frac{-3x}{x^2-9} + \frac{4}{2x-6}$ CD: $2(x-3)(x+3)$

$$\frac{(-3x)2}{2(x-3)(x+3)} + \frac{4(x+3)}{2(x-3)(x+3)} = \frac{-6x+4x+12}{2(x+3)(x-3)}$$

$$\frac{-2x+12}{2(x+3)(x-3)} = \frac{-2(x-6)}{2(x+3)(x-3)} = \boxed{\frac{-(x-6)}{(x+3)(x-3)} \quad x \neq \pm 3}$$

Example 8: $\frac{2x}{x^2-x-2} - \frac{4x}{x^2-3x+2}$ CD: $(x-2)(x+1)(x-1)$

$$\frac{2x(x-1)}{(x-1)(x-2)(x+1)} + \frac{-4x(x+1)}{(x-2)(x-1)(x+1)} = \frac{2x^2-2x-4x^2-4x}{(x+1)(x-1)(x-2)}$$

$$\frac{-2x^2-6x}{(x+1)(x-1)(x-2)} = \boxed{\frac{-2x(x+3)}{(x+1)(x-1)(x-2)} \quad x \neq \pm 1, 2}$$

Example 9: $\frac{5x}{x^2-x-6} - \frac{4}{x^2+4x+4}$ CD: $(x+2)^2(x-3)$

$$\frac{5x(x+2)}{(x+2)(x-3)(x+2)} + \frac{-4(x-3)}{(x+2)(x+2)(x-3)} = \frac{5x^2+10x-4x+12}{(x+2)^2(x-3)}$$

$$\boxed{\frac{5x^2+6x+12}{(x+2)^2(x-3)} \quad x \neq -2, 3}$$

Example 10: $\frac{x}{x-1} + \frac{2x-1}{x^2-3x+2}$ CD: $(x-1)(x-2)$

$$\frac{x(x-2)}{(x-1)(x-2)} + \frac{2x-1}{(x-2)(x-1)} = \frac{x^2-2x+2x-1}{(x-1)(x-2)}$$

$$= \frac{x^2-1}{(x-1)(x-2)} = \frac{(x-1)(x+1)}{(x-1)(x-2)} = \boxed{\frac{x+1}{x-2} \quad x \neq +1, +2}$$

Simplifying Complex Fractions

- Multiply the top fraction by the reciprocal (flip) of the denominator fraction

Example 11: $\frac{\frac{2}{x}}{\frac{1}{y}}$ = $\frac{2}{x} \cdot \frac{y}{1} = \frac{2y}{x}$

$x \neq 0$
 $y \neq 0$

Example 12: $\frac{\frac{3}{x+y}}{\frac{4}{1}}$ = $\frac{3}{x+y} \cdot \frac{1}{4} = \frac{3}{4(x+y)}$

Example 13: $\frac{\frac{3}{\frac{2}{x}+y}}{x}$ = $\frac{3}{\frac{2+yx}{x}} = \frac{3}{1} \cdot \frac{x}{2+yx}$

$$\frac{2}{x} + \frac{yx}{1x} = \frac{2+yx}{x}$$

$$= \boxed{\frac{3x}{xy+2}}$$

Example 14: $\frac{\frac{3}{x-4}}{1-\frac{2}{x-4}}$ = $\frac{3}{x-4} \cdot \frac{x-4}{x-4-2} = \frac{3}{x-6}$

$x \neq 4, 6$