Math 3

4.9 Writing Polynomial Equations

Unit 4

SWBAT write polynomial equations with given real and complex roots.

Roots	as	Factors
$x = a$	→	$(x - a) = 0$

When solving a polynomial function, oftentimes we can factor and set our factors equal to zero in order to solve. When writing a polynomial function, work this process backwards.

13
5 3
8

Solving Polynomial Equations:
Solve $5x^2 + 8x + 3 = 0$ by factoring.

$(5x^2 + 5x) + (3x + 3) = 0$

$5x(x+1) + 3(x+1) = 0$

$(5x+3)(x+1) = 0$

$5x+3=0$ $\quad x+1=0$

$x=-3/5$ $\quad x=-1$

Writing Polynomial Equations:
Write a polynomial function with roots at -1 and -3/5.

$x=-1$ $\quad x=-3/5$

$x+1=0$ $\quad 5x=-3$

$(x+1)(5x+3)=0$

$5x^2+3x+5x+3=0$

$5x^2+8x+3=0$

Example 1: Write the polynomial function with roots at 0, $-\frac{1}{4}$.

$x=-\frac{1}{4}$ $\quad x=0$

$4x+1=0$ $\quad x=0$

$x(4x+1)=0$

$4x^2+4x=0$

Example 2: Write the polynomial function with roots at $\sqrt{2}$, $-\sqrt{2}$, and $\frac{1}{3}$.

$x=\sqrt{2}$ $\quad x=-\sqrt{2}$ $\quad 3x=1$

$(x-\sqrt{2})(x+\sqrt{2})(3x-1)=0$

$(x^2+x\sqrt{2}-x\sqrt{2}-2)(3x-1)=0$

$(x^2-2)(3x-1)=0$

$3x^3-x^2-6x+2=0$

Example 3: Write a polynomial function with roots at 5 and $\pm 3i$.

$x=5$ $\quad x=-3i$ $\quad x=3i$

$x-5=0$ $\quad x+3i=0$ $\quad x-3i=0$

$(x-5)(x+3i)(x-3i)=0$

$(x-5)(x^2-3xi+3xi-9i^2)$

$(x-5)(x^2+9)=0$

$x^3-5x^2+9x-45=0$

You Try! Write a cubic function with zeros at -7 and $\pm 2i$.

$x=-7$ $\quad x=2i$ $\quad x=-2i$

$(x+7)(x-2i)(x+2i)=0$

$(x+7)(x^2+2xi-2xi-4i^2)=0$

$(x+7)(x^2+4)=0$

$x^3+7x^2+4x+28=0$

Example 4: Write a cubic equation with roots at $\frac{2}{3}$ and $2 \pm 3i$.

$3x = 2$ $\quad x = 2+3i$ $\quad x = 2-3i$

$3x-2=0$ $\quad x-2-3i=0$ $\quad x-2+3i=0$

$(3x-2)(x-2-3i)(x-2+3i)=0$

	x	-2	$-3i$
x	x^2	$-2x$	$-3xi$
-2	$-2x$	$+4$	$+6i$
$+3i$	$3xi$	$-6i$	$+9$

$(3x-2)(x^2-4x+13) = 3x^3-12x^2+39x-2x^2+8x-26$

$f(x) = 3x^3-14x^2+47x-26$

You Try! Write a cubic equation with roots at $\frac{1}{5}$ and $-3 \pm 2i$.

$5x = 1$ $\quad x = -3+2i$ $\quad x = -3-2i$

$5x-1=0$ $\quad x+3-2i=0$ $\quad x+3+2i=0$

$(5x-1)(x+3-2i)(x+3+2i)=0$

$(5x-1)(x^2+6x+13)$

$5x^3+30x^2+65x-x^2-6x-13$

$5x^3+29x^2+59x-13$

	x	$+3$	$-2i$
x	x^2	$+3x$	$-2xi$
$+3$	$3x$	$+9$	$-6i$
$+2i$	$2xi$	$+6i$	$+4$

$f(x) = 5x^3+29x^2+59x-13$

5.	Function: $f(x) = -x^5+x^4+5x^3+3x^2$

End Behavior:
as $x \to -\infty$, $f(x) \to \infty$
as $x \to \infty$, $f(x) \to -\infty$

Roots (with Multiplicity):
(3, 0) m:1
(-1, 0) m:2
(0, 0) m:2

$(x-3)(x+1)(x+1)(x^2)$

$(x^2-2x-3)(x^3+x^2)$

$x^5+x^4-2x^4-2x^3-3x^3-3x^2$

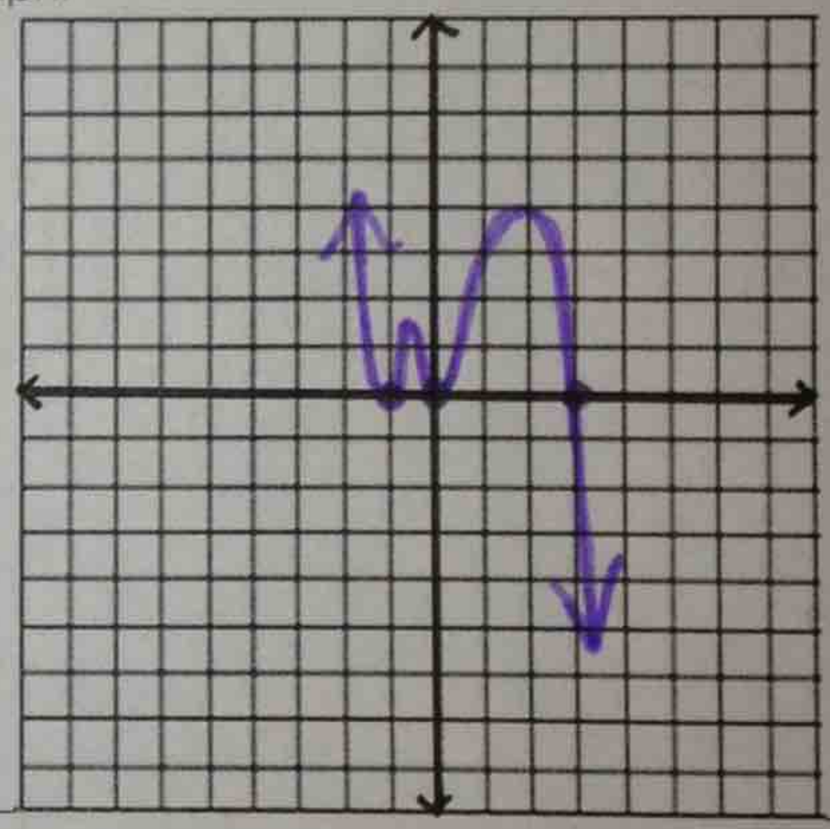
$-1(x^5-x^4-5x^3-3x^2)$

Value of the leading coefficient: -1

Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

Graph:



6.	Function: $f(x) = x^4-8x^2+16$

End Behavior:
as $x \to -\infty$, $f(x) \to \infty$
as $x \to \infty$, $f(x) \to \infty$

Roots (with Multiplicity):
(-2, 0) M: 2
(2, 0) M: 2

Value of the leading coefficient: 1

$(x-2)(x-2)(x+2)(x+2)$

$(x^2-4x+4)(x^2+4x+4)$

$x^4+4x^3+4x^2$
$-4x^3-16x^2-16x$
$+4x^2+16x+16$

$x^4+0x^3-8x^2+0x+16$

Domain: $(-\infty, \infty)$

Range: $[0, \infty)$

Graph: