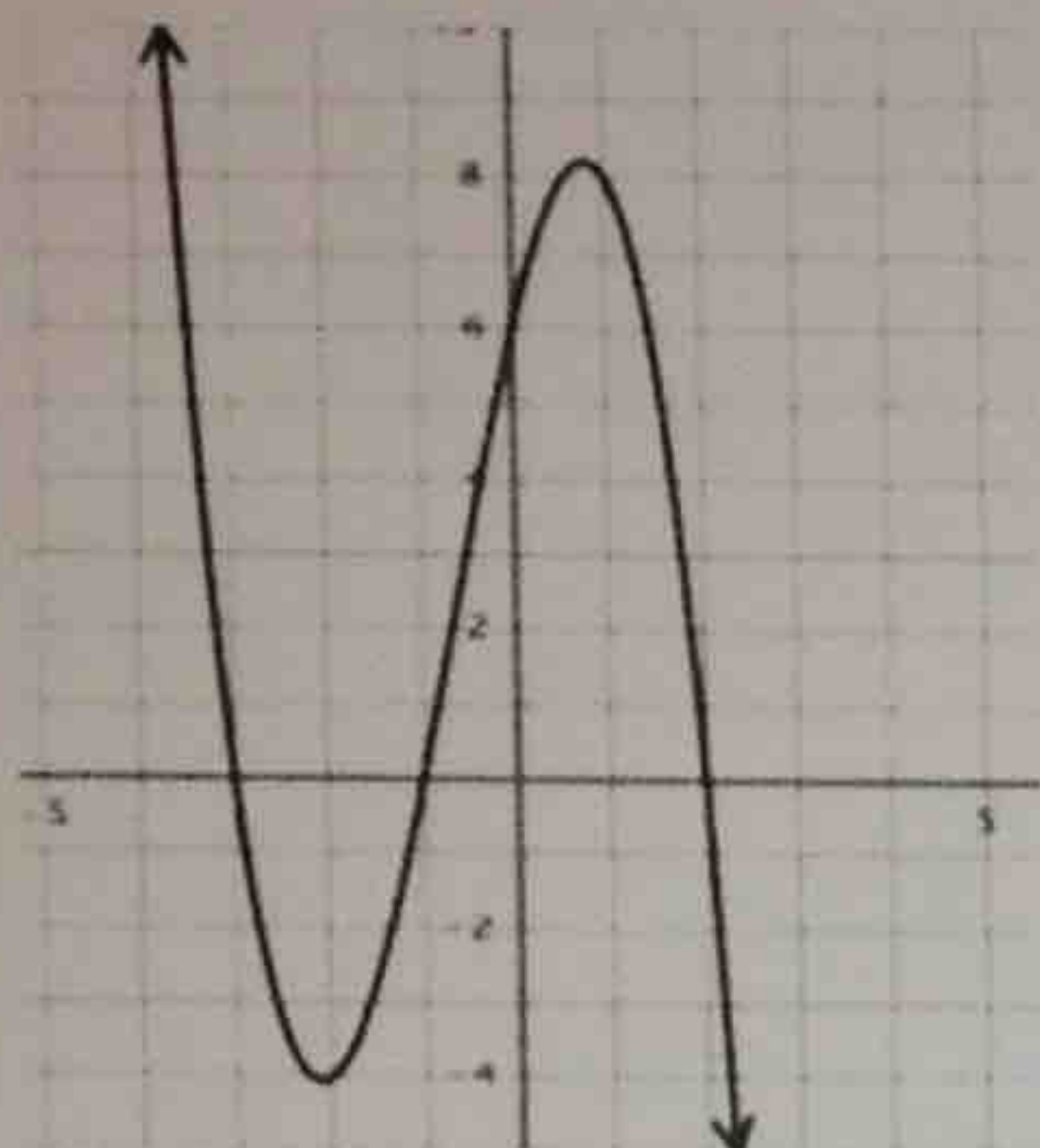


4.8 Solving Polynomials

SWBAT apply their knowledge of the Fundamental Theorem of Algebra to find the factors and roots of polynomials.



Example 1: Write everything you know about the following polynomial.

In case this was not part of what you wrote, use function notation to highlight values of importance for this function (i.e. $f(0) = 6$).

Example 2: Without using a calculator, find all linear factors and then determine all roots of the function. Sketch a graph with this information.

a) $f(x) = x^3 + 3x^2 - 4x - 12$

$x^2(x+3) - 4(x+3)$
 $(x^2-4)(x+3)$
 $(x-2)(x+2)(x+3)$

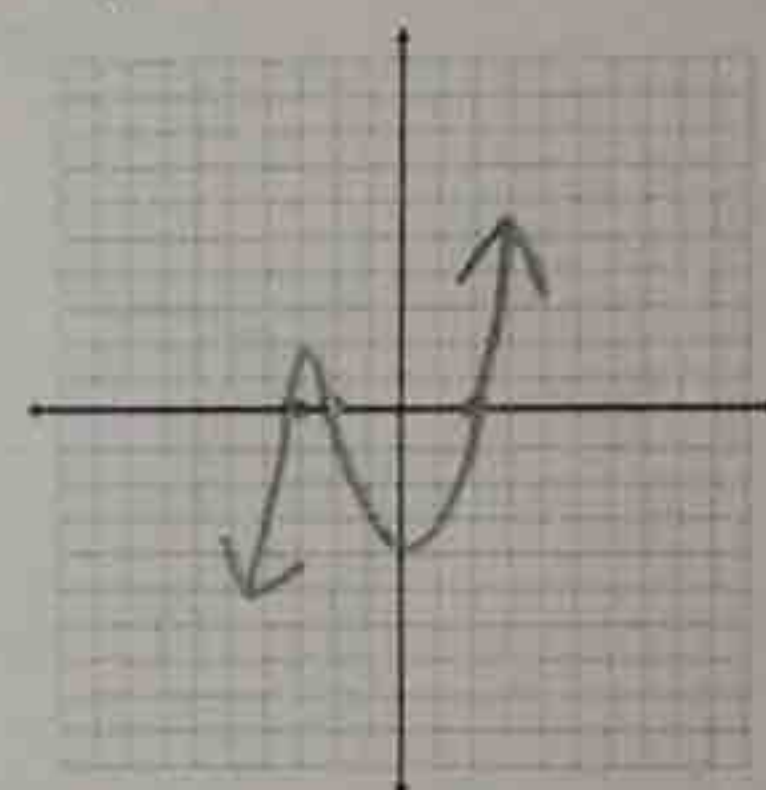
Factors:

$(x-2)$
 $(x+2)$
 $(x+3)$

Roots:

$\{2, -2, -3\}$

Graph:



b) $f(x) = x^3 - 3x^2 + x - 3$

$x^2(x-3) + 1(x-3)$
 $(x^2+1)(x-3)$
 $(x+i)(x-i)(x-3)$

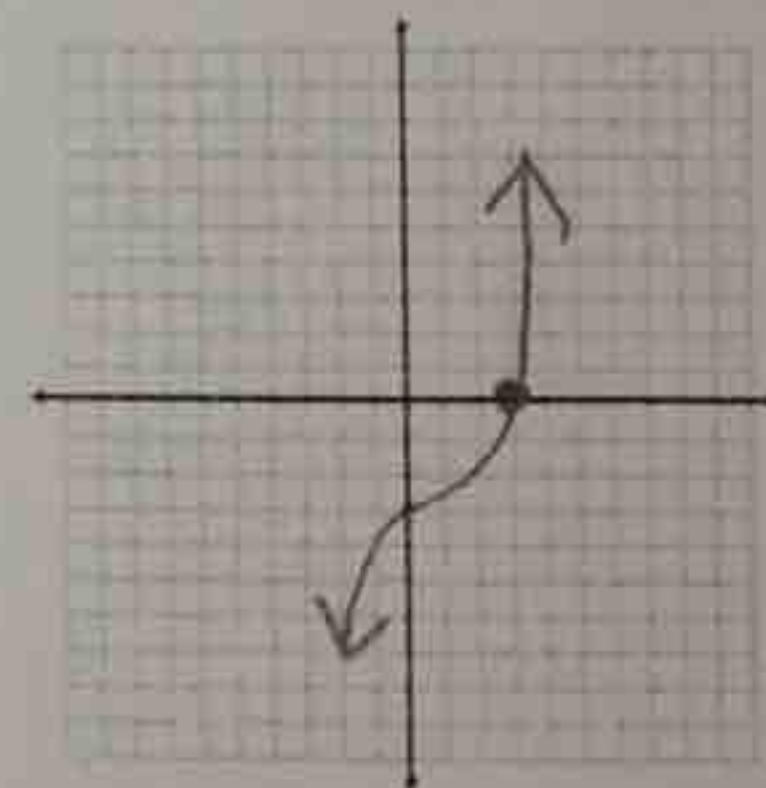
Factors:

$(x+i)$
 $(x-i)$
 $(x-3)$

Roots:

$\{\pm i, 3\}$

Graph:



c) $f(x) = x^3 + x^2 + 4x + 4$

$x^2(x+1) + 4(x+1)$
 $(x^2+4)(x+1)$
 $(x+2i)(x-2i)(x+1)$

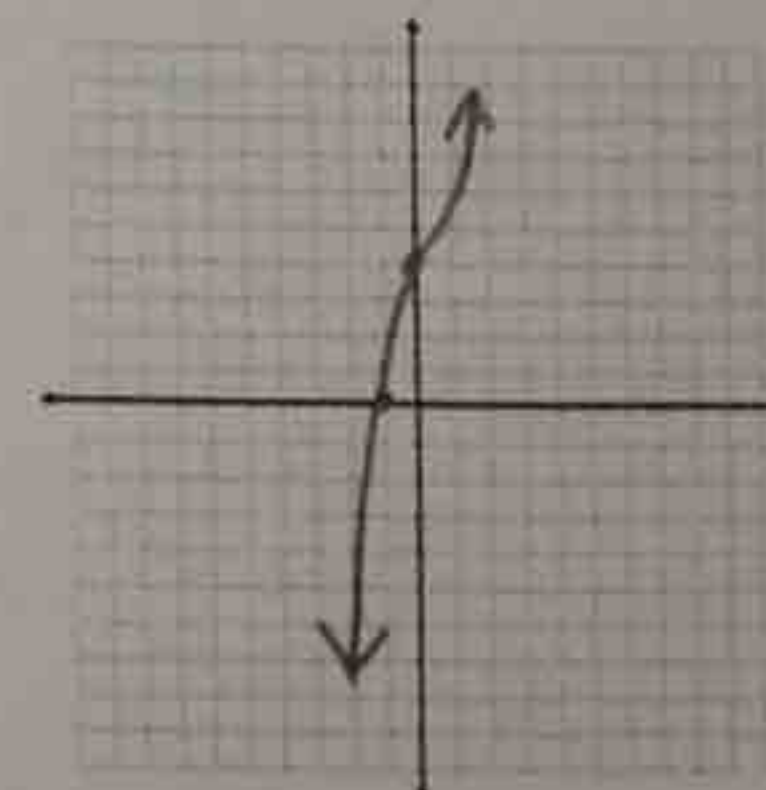
Factors:

$(x+2i)$
 $(x-2i)$
 $(x+1)$

Roots:

$\{\pm 2i, -1\}$

Graph:



d) $f(x) = x^5 - 8x^3 - 9x$

$x(x^4 - 8x^2 - 9)$
 $x^3(x^2-9) - 9x$
 $x^3(x^2-9) - 9x$
 $(x^3+x)(x^2-9)$
 $x(x^2+1)(x+3)(x-3)$
 $x(x+i)(x-i)(x+3)(x-3)$

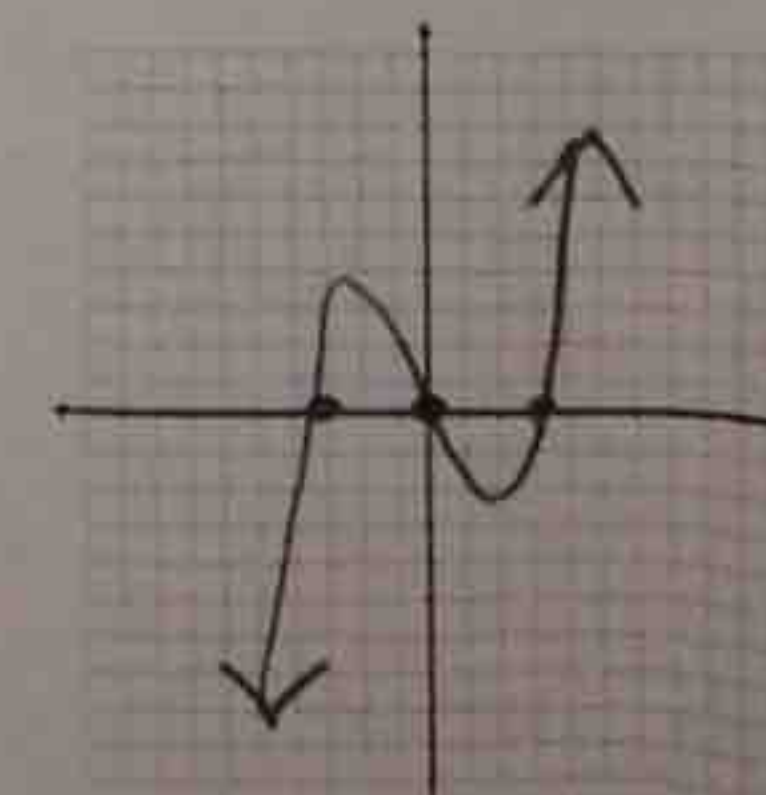
Factors:

x
 $(x+i)$
 $(x-i)$
 $(x+3)$
 $(x-3)$

Roots:

$\{0, \pm i, \pm 3\}$

Graph:



Example 3: Using the given factor, find all remaining factors and then determine all roots of the function. Sketch a graph with this information.

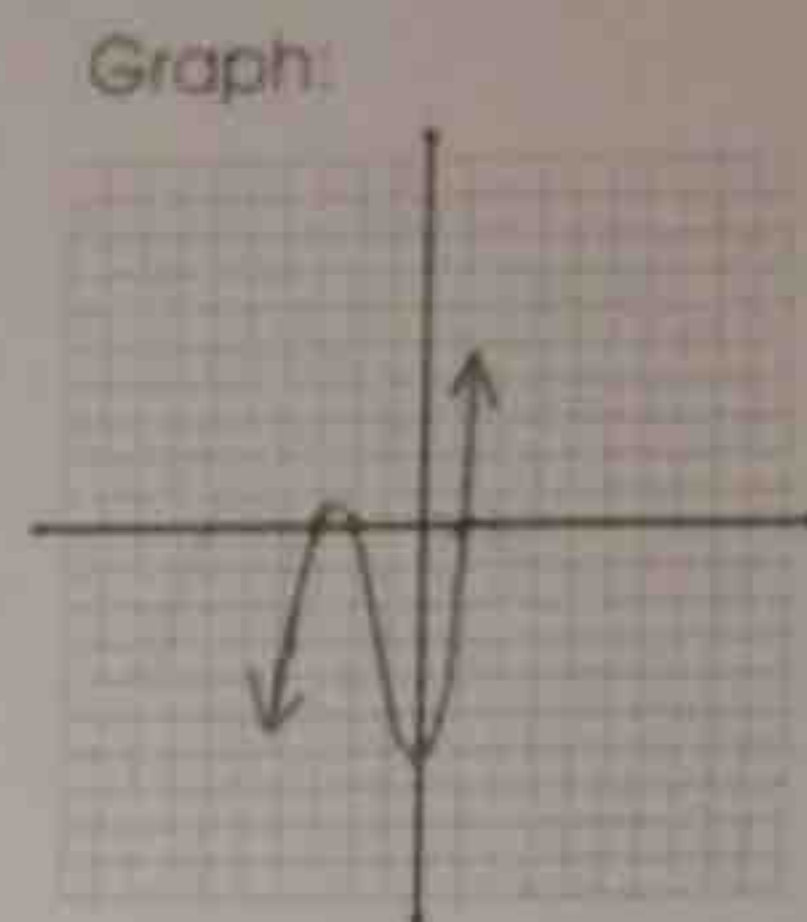
a) $f(x) = x^3 + 4x^2 + x - 6$

$$\begin{array}{r} -3 \overline{) 1 \quad +4 \quad 1 \quad -6} \\ \underline{\downarrow -3 \quad -3 \quad 6} \\ 1 \quad 1 \quad -2 \quad \boxed{0} \end{array}$$

$x^2 + x - 2$
 $(x+2)(x-1)$

Factor: $(x+3)$
 $(x+2)$
 $(x-1)$

Roots: $\{-3, -2, 1\}$



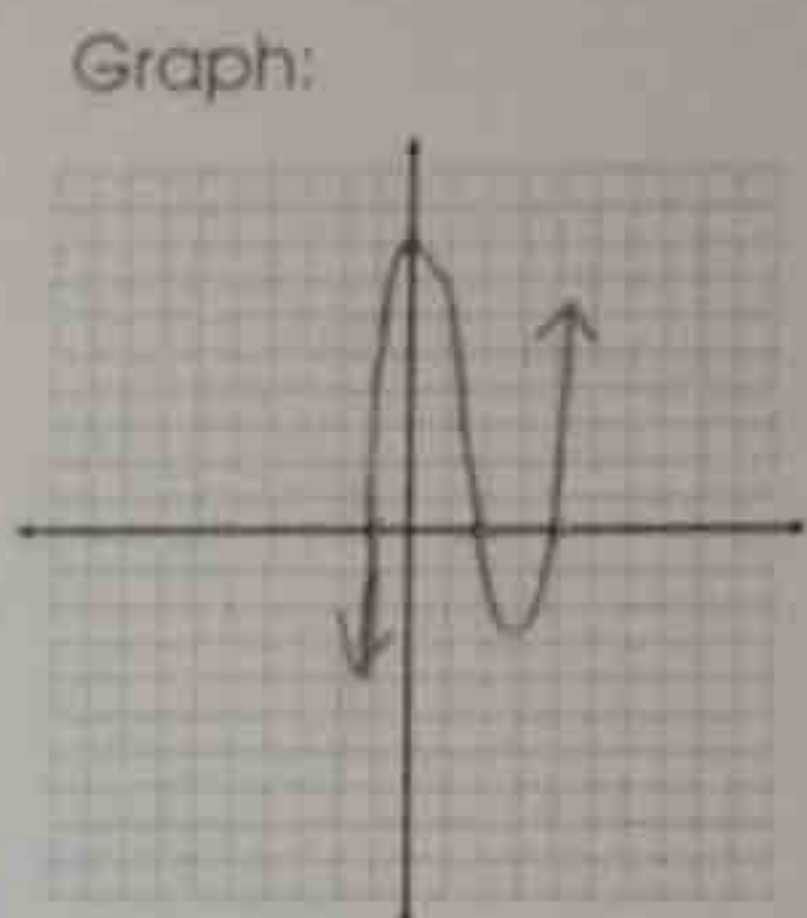
b) $f(x) = x^3 - 5x^2 + 2x + 8$

$$\begin{array}{r} 2 \overline{) 1 \quad -5 \quad 2 \quad 8} \\ \underline{\downarrow 2 \quad -6 \quad -8} \\ 1 \quad -3 \quad -4 \quad \boxed{0} \end{array}$$

$x^2 - 3x - 4$
 $(x-4)(x+1)$

Factor: $(x-2)$
 $(x-4)$
 $(x+1)$

Roots: $\{2, 4, -1\}$



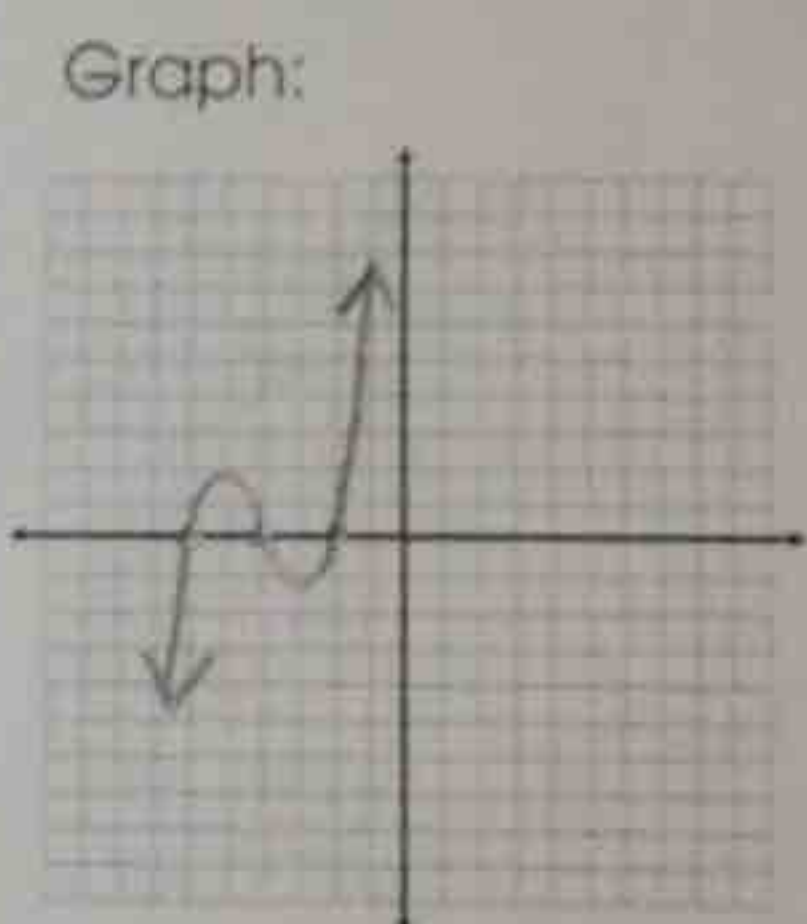
c) $f(x) = x^3 + 6x^2 + 11x + 6$

$$\begin{array}{r} -1 \overline{) 1 \quad 6 \quad 11 \quad 6} \\ \underline{\downarrow -1 \quad -5 \quad -6} \\ 1 \quad 5 \quad 6 \quad \boxed{0} \end{array}$$

$x^2 + 5x + 6$
 $(x+2)(x+3)$

Factor: $(x+1)$
 $(x+2)$
 $(x+3)$

Roots: $\{-1, -2, -3\}$



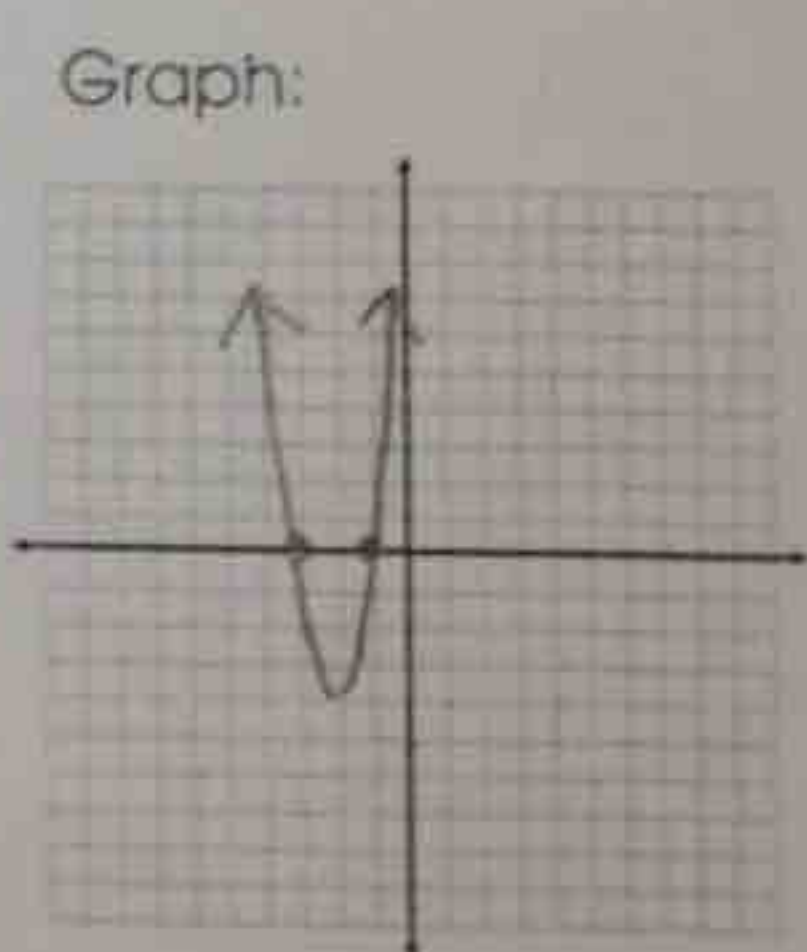
d) $f(x) = x^4 + 4x^3 + 7x^2 + 16x + 12$

$$\begin{array}{r} -1 \overline{) 1 \quad 4 \quad 7 \quad 16 \quad 12} \\ \underline{\downarrow -1 \quad -3 \quad -4 \quad -12} \\ 1 \quad 3 \quad 4 \quad 12 \quad \boxed{0} \end{array}$$

$(x^3 + 3x^2 + 4x + 12)$
 $x^2(x+3) + 4(x+3)$
 $(x+2i)(x-2i)(x+3)$

Factor: $(x+1)$
 $(x+2i)$
 $(x-2i)$
 $(x+3)$

Roots: $\{-1, -3, \pm 2i\}$



e) $f(x) = x^4 - 4x^3 + x^2 + 12x - 12$

$$\begin{array}{r} 2 \overline{) 1 \quad -4 \quad 1 \quad 12 \quad -12} \\ \underline{\downarrow 2 \quad -4 \quad -6 \quad 12} \\ 1 \quad -2 \quad -3 \quad 6 \quad \boxed{0} \end{array}$$

$(x^3 - 2x^2 - 3x + 6)$
 $x^2(x-2) - 3(x-2)$
 $(x^2-3)(x-2)$

Factor: $(x-2)$
 $(x-2)$
 (x^2-3)
 or
 $(x-\sqrt{3})$
 $(x+\sqrt{3})$

Roots: $\{\pm\sqrt{3}, 2\}$

