Math 3

4.7 Remainder & Factor Theorems

Unit 4

SWBAT use the remainder and factor theorems to determine if a factor is a root of a polynomial.

Remainder Theorem: When we divide a polynomial $f(x)$ by a divisor $(x - c)$, the remainder, r, equals $f(c)$.

Using Synthetic Division:	Using the Remainder Theorem:
What is the remainder when $2x^2 - 5x - 1$ is divided by $x - 3$? 3 | 2 -5 -1 ↓ 6 3 2 1 |2| R = 2	What is the remainder when $2x^2 - 5x - 1$ is divided by $x - 3$? $f(3) = 2(3)^2 - 5(3) - 1$ $f(3) = 18 - 15 - 1$ $f(3) = 2$ R = 2

Example 1: Use the remainder theorem to determine the remainder of the following division problems.

a) $(p^4 + 5p^3 - 11p^2 - 25p + 29) \div (p + 6)$

$f(-6) = (-6)^4 + 5(-6)^3 - 11(-6)^2 - 25(-6) + 29$

$f(-6) = -1$

b) $(3x^3 - 4x^2 - 17x + 6) \div (3x - 1)$

$f(1/3) = 3(1/3)^3 - 4(1/3)^2 - 17(1/3) + 6$

$f(1/3) = 0$

c) $(y^4 - 8y^3 + 10y^2 + 2y + 4) \div (y - 2)$

$f(2) = (2)^4 - 8(2)^3 + 10(2)^2 + 2(2) + 4$

$f(2) = 0$

d) $(4v^3 + 6v^2 - 8v - 12) \div (2v - 3)$

$f(3/2) = 4(1.5)^3 + 6(1.5)^2 - 8(1.5) - 12$

$f(3/2) = 3$

Factor Theorem: If we calculate $f(c)$ and it equals 0, that means the remainder is 0, and $(x - c)$ must be a factor of the polynomial

- Knowing that x – c is a factor is the same thing as knowing that c is a root (and vice versa).
- The factor "x – c" and the root "c" are the same thing

Example 2: Determine whether each binomial is a factor of $(x^3 + 4x^2 + x - 6)$

a) $x + 1$

$f(-1) = (-1)^3 + 4(-1)^2 + (-1) - 6$

$f(-1) = -4$ Not a factor

b) $x + 3$

$f(-3) = (-3)^3 + 4(-3) + (-3) - 6$

$f(-3) = 0$ $x + 3$ is a factor!

Example 3: Consider the polynomial $P(x) = x^3 + kx^2 + x + 6$. Find the value of k so that x + 2 is a factor of P.

$(-2)^3 + K(-2)^2 + (-2) + 6 = 0$

$-8 + 4K + 4 = 0$

$4K - 4 = 0$

$4K = 4$

$K = 1$

Check: $f(-2) = (-2)^3 + 1(-2)^2 + (-2) + 6$

$f(-2) = 0$

$K = 1$

Example 4: When you divide $P(x) = x^3 + 4x^2 - 2x + k$ by $(x - 3)$, the remainder is -2. What is the remainder of P(x) when you divide by $(x + 2)$?

$(3)^3 + 4(3)^2 - 2(3) + K = -2$

$57 + K = -2$

$K = -59$

$f(-2) = (-2)^3 + 4(-2)^2 - 2(-2) - 59$

$= -47$

R = -47