3.3 Properties of Logarithms

SWBAT condense, expand, and solve logarithmic equations by using the properties of logarithms.

Since logarithms are inverses of exponentials the properties of logarithms can be derived from the properties of exponents.

- When **expanding** logs, we want to have multiple logs being added or subtracted from each other.
- When **condensing** logs, we want to end with one log and multiple variables.

PRODUCT PROPERTY:

$$\log_b mn = \log_b m + \log_b n$$

The logarithm of a product (multiplication) is the same as the addition of the logs

Example 1: Expand $\log_3 (4)(7)$

$\log_3 4 + \log_3 7$

You Try! Expand $\log xyz =$

$\log x + \log y + \log z$

Example 2: Condense $\log_6 10 + \log_6 x + \log_6 7$

$\log_6 (70x)$

You Try! Condense: $\log_4 x + \log_4 m + \log_4 9$

$\log_4 9xm$

QUOTIENT PROPERTY:

$$\log_b \frac{m}{n} = \log_b m - \log_b n$$

The logarithm of a quotient (division) is the same as the subtraction of the logs

Example 3: Expand: $\log_4 \frac{x}{y} =$

$\log_4 x - \log_4 y$

You Try! Expand $\log_7 \frac{5}{9} =$

$\log_7 5 - \log_7 9$

Example 4: Condense $\log_4 10 - \log_4 x$

$\log_4 \frac{10}{x}$

You Try! Condense $\log_2 x - \log_2 m$

$\log_2 \frac{x}{m}$

EXPONENT PROPERTY:

$$\log_b m^p = p \log_b m$$

The logarithm of the power is the same as the multiplication of the logs

Example 5: Expand $\log_5 x^4 =$

$4 \log_5 x$

You Try! Expand $\log_{47} y^{\frac{1}{3}} =$

$\frac{1}{3} \log_{47} y$

Example 6: Condense $5 \log_{12} x =$

$\log_{12} x^5$

You Try! Condense $\frac{7}{4} \log_9 z =$

$\log_9 z^{7/4}$

Square Roots: If you ever see a square root in a problem, you must convert it to a rational exponent. *Remember – the index becomes the denominator of the exponent!

Example 7: Expand $\log_9 \sqrt{8z} =$

$\log_9 (8z)^{\frac{1}{2}}$

$= \frac{1}{2}(\log_9 8 + \log_9 z)$

$= \frac{1}{2}\log_9 8 + \frac{1}{2}\log_9 z$

You Try! Expand $\log_{12} \sqrt[4]{\frac{x}{z}} =$

$\log_{12} \left(\frac{x}{z}\right)^{1/4}$

$\frac{1}{4}\log_{12} x - \frac{1}{4}\log_{12} z$

Example 8: Condense: $\frac{5 \log x}{4} =$

$\frac{\log x^5}{4} = \frac{1}{4}\log x^5$

$= \log(x^5)^{1/4} = \log \sqrt[4]{x^5}$

Putting it all together: Expand or condense the following logarithms using properties listed above.

a) Expand $\log_3 x^5 y^7 =$

$= \log_3 x^5 + \log_3 y^7$

$= 5\log_3 x + 7\log_3 y$

b) Expand $\log_5 \frac{a^3}{b^7} =$

$= \log_5 a^3 - \log_5 b^7$

$= 3\log_5 a - 7\log_5 b$

c) Expand $\log_5 \frac{g^6 h^2}{k^5} =$

$(6\log_5 g + 2\log_5 h) - (5\log_5 k)$

d) Condense $5\log_2 x + 7\log_2 y$

$\log_2 x^5 y^7$

e) Condense $6\log_5 g - 9\log_5 b$

$\log_5 \frac{g^6}{b^9}$

f) Condense $7\log_4 x + \log_4 y - 6\log_4 z$

$\log_4 \frac{x^7 y}{z^6}$

RULE: If there are multiple logs on the same side of an equation, CONDENSE before solving, then apply the rules from Day 1 (cancel, swoosh, or evaluate)

Example 9: $\log_3 5 - \log_3 x = \log_3 10$

$\cancel{\log_3} \frac{5}{x} = \cancel{\log_3} 10$

$\frac{5}{x} = 10$ $\quad x = \frac{1}{2}$

Example 10: $4\log_2 x + \log_2 5 = \log_2 405$

$\cancel{\log_2} 5x^4 = \cancel{\log_2} 405$

$5x^4 = 405$

$x^4 = 81$

$x = 3$

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1. Expand $\log_3 \sqrt{\frac{x^5 y^6}{z^7}} =$

$\left(\frac{5\log_3 x}{2} + \frac{6\log_3 y}{2}\right) - \frac{7\log_3 z}{2}$

2. Condense $4\log_2 c + 8\log_2 d - \log_2 e$

$\log_2 \frac{c^4 d^8}{e}$

3. Solve for x: $3\log_5 x - \log_5 4 = \log_5 16$

$\cancel{\log_5} \frac{x^3}{4} = \cancel{\log_5} 16$

$\frac{x^3}{4} = 16$ $\quad x^3 = 64$

$x = 4$

4. Solve for x: $\cancel{\log} a = \cancel{\log}(4a - 9)$

$a = 4a - 9$

$-3a = -9$

$a = 3$

5. Solve for x: $\cancel{\log}(-3m - 1) = \cancel{\log}(-4m - 6)$

$-3m - 1 = -4m - 6$

$m - 1 = -6$

$m = -5$

6. Solve for x: $5\log_{19} 2 - \log_{19} x = \log_{19} 8$

$\cancel{\log_{19}} \frac{2^5}{x} = \cancel{\log_{19}} 8$

$\frac{32}{x} = 8$ $\quad x = \frac{32}{8} = 4$