**NC Math 3 Pacing Guide**

This pacing guide is the collaborative work of math teachers, coaches, and curriculum leaders from 38 NC public school districts. The teams worked through two face-to-face meetings and digitally to compile the information presented. NC Math 1, 2, and 3 standards were used to draft possible units of study for these courses. This is a first draft living document. Teams plan to meet throughout the year to continually tweak, update and refine these guides. Updates will be posted as available to this google document.

Please reference the [NC Math 1, 2, or 3 standards](http://maccss.ncdpi.wikispaces.net/HS+Standards) for any questions or discrepancies. This document should be used only **after** reading the NC Math 1, 2, and 3 standards and instructional guides provided by NC DPI.

If you have suggestions or comments that you would like the collaborative writing team to consider for revisions, please email [sdupree@wcpss.net](mailto:sdupree@wcpss.net) or [stefanie.buckner@bcsemail.org](mailto:stefanie.buckner@bcsemail.org).

**Learning Intentions:** These are big ideas, understandings, important math that needs to be developed. They are not necessarily measurable statements. Ideally a unit will have a handful of learning intentions.

**Success Criteria:** These are directly associated with a learning intention and articulate to students measurable, tangible, observable demonstrations of the learning intention. Typically one learning intention has around 3 to 5 success criteria.

|  |  |  |
| --- | --- | --- |
| Unit: **FUNCTIONS AND THEIR INVERSES**  Suggested Order: 1 of 8  Suggested Time: 10 days semester block (90-minute classes) | | |
| **Rationale:** This first unit builds upon students’ previous work with modeling functions in Math 1 and Math 2. This unit helps students transition from modeling in the real world to more abstract mathematical concepts like polynomial and rational functions. It develops the notion of the inverse function of quadratic, exponential, and linear functions and introduces piecewise-defined and absolute value functions through multiple representations, i.e. graphing, equations, tables, verbal descriptions, etc. Since students in Math 1 and Math 2 have already worked with linear, quadratic, and exponential functions, this allows teachers a chance to begin with content that is familiar to students. It also assists teachers in identifying misconceptions, obstacles, and gaps in prior learning. | | |
| Overarching Standards - NC.M3.A-SSE.1b, NC.M3.A-SSE.2, NC.M3.A-REI.1, NC.M3.F-BF.1b | | |
| *Major Work* | | |
| Standards | Learning Intentions | Success Criteria |
| NC.M3.F-IF.4  NC.M3.F-IF.9  NC.M3.F-IF.2  NC.M3.F-IF.7  NC.M3.F-BF.1b  NC.M3.F-BF.3 | A. Compare functions using multiple representations of and understand key features to interpret, analyze, and find solutions. | A1. I can compare key features of two different functions each with unique representations such as symbols, graphs, tables, or with verbal descriptions.  A2. I can use function notation to evaluate piecewise-defined functions for inputs in their domains.  A3. I can build a new function, in a real life situation, by combining standard function “types” using arithmetic operations.  A4. I can make sense of a function by extending it graphically or in a table.  A5. I can understand the effects on a graph through transformations graphically. |
| NC.M3.F-BF.4  NC.M3.F-BF.4a  NC.M3.F-BF.4b  NC.M3.F-BF.4c | B. Understand inverse relationships, describe them algebraically, and use these relationships to solve, analyze and interpret. | B1. I can find the inverse of a function (symbolically, graphically, numerically and in tables, and by verbal description).  B2. I can recognize the inverse relationships between exponential and logarithmic, quadratic and square root, and linear to linear functions.  B3. I can use inverse relationships in real world situations to interpret and solve.  B4. I can determine if an inverse function exists by analyzing a table, a graph, or an equation.  B5. I can use function notation to represent inverses. |
| NC.M3.A-CED.1  NC.M3.A-CED.2  NC.M3.A-CED.3  NC.M3.A-SSE.1  NC.M3.A-REI.11 | C. Understand and recognize piecewise defined relationships. Students can solve absolute value equations and inequalities. Students can solve systems of equations that include piecewise defined relationships (including absolute value) and can use a graphing tool when necessary. Students can identify key parts in expressions and equations. | C1. I can create equations and inequalities in one and two variables that represent absolute value and piecewise defined relationships.  C2. I can use absolute value equations and inequalities in one variable to solve problems algebraically and graphically.  C4. I can create systems of equations and/or inequalities to model situations in context.  C5. I can identify and interpret parts of piecewise-defined and absolute value expressions.  C6. I can use technology to solve equations through multiple representations (tables and graphs). |
| **Resources:** <http://www.mathematicsvisionproject.org/uploads/1/1/6/3/11636986/sec3_mod1_funinv_se_31213.pdf>  [The Fire Station Problem](http://www.txar.org/training/materials/Algebra_II/10MAPAbsoluteValueStudentLesson03142007.pdf) - modeling with absolute value through context | | |
| **Vocabulary:** Inverse, relation, function, one-to-one, logarithm, horizontal line test, vertical line test, Absolute Value, Piecewise function, system of equations, domain, range, x-intercept, y-intercept, increasing/decreasing intervals), evaluate, end behavior | | |
| **Possible Honors Topics:** Greatest Integer Function | | |

|  |  |  |
| --- | --- | --- |
| Unit: **EXPONENTIAL AND LOGARITHMIC FUNCTIONS**  Suggested Order: 2 of 8  Suggested Time: 9 days on a semester schedule (90-minute classes) | | |
| **Rationale:** Following the functions unit, this unit continues to build upon familiarity with exponents and exponential functions and introduces logarithmic functions. Additionally, solving exponential and logarithmic equations involves using algebraic operations students have practiced in Math 1 and Math 2, thus this unit seeks to build continued opportunities for students to be successful at the beginning of Math 3. Furthermore, flexibility with exponential and logarithmic models is essential for competence in Precalculus and Calculus; therefore, teachers should stress a modeling approach to this unit. | | |
| Overarching Standards - NC.M3.A-SSE.1b, NC.M3.A-SSE.2, NC.M3.A-REI.1, NC.M3.F-BF.1b | | |
| *Major Work* | | |
| Standards | Learning Intentions | Success Criteria |
| NC.M3.A-CED.1  NC.M3.A-CED.2  MC.M3.A-SSE.1  NC.M3.A-SSE.3c | A. Understand how to create exponential equations and graphs with one or two variables, and be able to identify the different parts of an exponential equation and relate them to the real world. | A1. I can create and graph exponential equations with one or two variables and use them to solve problems algebraically.  A2. I can identify and interpret parts of an exponential expression and relate them to a real-world situation.  A3. I can use the properties of exponents to write equivalent forms of exponential expressions and state the domain. |
| NC.M3.F-BF.3  NC.M3.F-IF.4  NC.M3. F-IF.7  NC.M3. F-IF.9  NC.M3.F-BF.1a  NC.M3.F-BF.4  NC.M3.F-LE.4 | B. Recognize the relationship between exponential and logarithmic equations as inverses using multiple representations, interpret the key features of the graph, and use them to solve equations and model real world phenomena. | B1. I can identify and describe the effects of transformations of exponential and logarithmic equations in relation to their parent functions.  B2. I can graph and describe key features of exponential and logarithmic functions with and without technology.  B3. I can use different representations to compare key features of logarithmic and exponential functions and interpret them in context.  B4. I can build exponential functions to represent relevant relationships/quantities given a real-world situation or mathematical process.  B5: I can find and apply the inverse relationship between exponential and logarithmic equations through different representations.  B6: I can determine if the inverse function exists through different representations and use it to solve real-world problems.  B7: I can use logarithms to solve exponential models. |

|  |
| --- |
| Resources for Exponential/Logarithmic Unit:  [Logarithmic & Exponential Problem Solving](https://drive.google.com/open?id=0B-i_U7CppmLhSnRXY3FBUUxIUVE)  <http://www.doe.virginia.gov/testing/solsearch/sol/math/AFDA/m_ess_afda-1~-2~-3~-4_loga.pdf>  <http://www.carlisleschools.org/webpages/wolfer/files/lab%20modeling%20with%20mms.pdf> |
| **Vocabulary:** Domain, range, Logarithm, natural logarithm, exponential function, compound interest, base, growth/decay factor, transformation(s), rate, principal, increasing/decreasing intervals, x-intercept, y-intercept, end behavior |
| **Possible Honors Topics:** Properties of Logarithms, Use exponents to solve logarithmic equations |

|  |  |  |
| --- | --- | --- |
| Unit: **POLYNOMIAL FUNCTIONS**  Suggested Order: 3 of 8  Suggested Time: 9 days on a semester schedule (90-minute classes) | | |
| **Rationale**: Students will begin by continuing their modeling work (connected to the first unit), with expressions or functions that represent familiar topics such as perimeter and area, and volume. Students have worked with quadratics in Math 1 and 2, so the model they create for area will be familiar to them. The modeling of volume would introduce a cubic polynomial and present the opportunity to begin exploring polynomials of higher degree more in depth. The placement of this unit before Modeling with Geometry is strongly suggested. | | |
| Overarching Standards - NC.M3.A-SSE.1b, NC.M3.A-SSE.2, NC.M3.A-REI.1, NC.M3.F-BF.1b | | |
| *Major Work* | | |
| Standards | Learning Intentions | Success Criteria |
| NC.M3.G-GMD.3  NC.M3.G-MG.1 | Students understand surface area and volume of geometric figures can be modeled by polynomial functions. | I can model the surface area and volume of a geometric figure with a polynomial function.\*  \**Proficiency is not expected for this success criteria until the next unit. However, this gives relevance to the study of polynomial functions.* |
| NC.M3.N-CN.9 | A. Recognize parts of a polynomial, and apply the Fundamental Theorem of Algebra to determine the types and number of solutions. | A1. I can identify the number of solutions to a polynomial as exactly the degree of the polynomial, including multiplicities, Real, and Complex solutions. |
| NC.M3.A-SSE.1  NC.M3.A-APR.2  NC.M3.A-APR.3  NC.M3.A-CED.1  NC.M3.A-CED.2  NC.M3.F-BF.1a | B. Understand and apply the Remainder Theorem, the Factor Theorem, and the Division Algorithm. Create polynomial equations in one or two variables and use them to solve problems algebraically and graphically. | B1: I can identify the constant, linear, quadratic and lead terms, lead coefficients and degree of a polynomial.  B2. I can divide polynomials by using long division and synthetic division.  B3. I can apply the remainder theorem.  B4. I can apply the factor theorem.  B5. I can create and solve one and two variable polynomial equations algebraically and graphically. |
| NC.M3.F-IF.4  NC.M3.F-IF.7  NC.M3.F-IF.9  NC.M3.F-BF.1a  NC.M3.F-BF.1b  NC.M3.F-BF.3  NC.M3.F-LE.3 | C. Recognize key features, zeros, and transformations of polynomial functions. Analyze a polynomial function and compare two or more functions by using their key features. Given solutions or a graph, write the equation of polynomial function. Graph transformations. Compare the relative rates of growth of exponential and polynomial functions. | C1. I can identify the key features of a polynomial function.  C2. I can compare key features of two given functions in a variety of representations.  C3. I can create a polynomial function given the zeros or a graph.  C4. I can determine whether an exponential or polynomial function increases more rapidly.  C5. I can use rules of transformations on polynomial functions.  C6. I can interpret key features of functions in context. |
| **Vocabulary:** Polynomial, leading coefficient, degree, term, factor, Fundamental Theorem of Algebra, domain, range, x-intercept, y-intercept, increasing/decreasing intervals, relative vs. absolute maximum(s) & minimum(s), end behavior, zero, root, synthetic division, long division, Remainder Theorem, Factor Theorem, complex number, imaginary number, multiplicity | | |

|  |  |  |
| --- | --- | --- |
| Unit: **MODELING WITH GEOMETRY**  Suggested Order: 4 of 8  Suggested Time: 7 days on a semester schedule (90-minute classes) | | |
| **Rationale**: This unit transitions from polynomial work to geometric concepts that require the use of algebra. It is intentionally placed after the polynomials unit because the polynomials unit is suggested to begin with geometric modeling that results in a polynomial. Teaching this unit right after the conclusion of polynomials, allows you to circle back to the geometric modeling concept and study it to its full depth. The placement of this unit also gives students a break from the *heavy* algebra work of polynomials prior to beginning rational functions. | | |
| Overarching Standards - NC.M3.A-SSE.1b, NC.M3.A-SSE.2, NC.M3.A-REI.1, NC.M3.F-BF.1b | | |
| *Major Work* | | |
| Standards | Learning Intentions | Success Criteria |
| NC.M3.G-GPE.1 | A. Derive the equation of a circle as well as distinguishing the center and radius of a circle from an equation. | A1: I can use the Pythagorean Theorem to derive the equation of a circle.  A2: I can complete the square to find the center and radius of a circle. |
| NC.M3.G-GMD.3  NC.M3.G-GMD.4  NC.M3.G-MG.1  NC.M3.G-CO.14 | B. Implement surface area and volume of geometric figures and model using polynomial functions. Furthermore, relating cross sections with two-dimensional and three-dimensional figures. | B1: I can use volume formulas to solve problems.  B2: I can identify the two-dimensional shape formed by the cross-section of a three dimensional figure.  B3. I can identify the three-dimensional shape formed by the rotation of a two-dimensional figure.  B4: I can use geometric concepts to model and solve real-world situations. |
| **Vocabulary:** Parallelogram, diagonal, median, altitude, angle bisector, perpendicular bisector, prism, cylinder, pyramid, cone, sphere, cross section, rotation | | |

|  |  |  |
| --- | --- | --- |
| Unit: **RATIONAL FUNCTIONS**  Suggested Order: 5 of 8  Suggested Time: 10 days on semester schedule (90-minute classes) | | |
| **Rationale**: This unit is intended to develop students’ understanding of rational functions. It is suggested to be taught in close proximity to the polynomials unit because of the connection of rational expressions to the division of polynomials. This unit should begin with reviewing both simplification of fractions and all arithmetic operations to help students understand the similarities and differences between rational numbers and expressions. | | |
| Overarching Standards - NC.M3.A-SSE.1b, NC.M3.A-SSE.2, NC.M3.A-REI.1, NC.M3.F-BF.1b | | |
| *Major Work* | | |
| Standards | Learning Intentions | Success Criteria |
| NC.M3.A-SSE.1  NC.M3.A-APR.6  NC.M3.A-APR.7  NC.M3.A-CED.1  NC.M3.A-CED.2  NC.M3.A-REI.2 | A. Recognize rational expressions as the division of two polynomials and use properties of simple fractions to analyze, perform arithmetic operations, create and solve equations that model real world phenomena. | A1. I can rewrite and simplify a rational expression by factoring, long division, or synthetic division.  A2. I can explain how operations on rational expressions are the same as simple fractions.  A3. I can multiply and divide rational expressions.  A4. I can find LCD in order to add and subtract rational expressions.  A5. I can recognize the difference between adding rational expressions and solving rational expressions.  A6. I can solve a one variable rational equation algebraically or using a graph.  A7. I can give examples showing how extraneous solutions may arise when solving rational equation.  A8. I can create and solve a rational equation to solve an application.  A9. I can interpret the terms, factors, and coefficients of rational expressions. |
| NC.M3.F-IF.4  NC.M3.F-IF.7  NC.M3.F-IF.9 | B. Understand and interpret the key features, uses and limitations of multiple representations of a rational function. | B1. I can determine the domain and discontinuities of a rational function by finding vertical asymptotes and holes of the function.  B2. I can determine the range of a rational function by finding the horizontal asymptotes and holes of a function.  B3. I can find the end behaviors of a rational function by looking at a graph or table.  B4. I can find x and y intercepts from a graph, while recognizing that they may not exist. |
| **Vocabulary:** Rational function, common denominator, complex fraction, proportion, extraneous solution, domain restriction, domain, range, x-intercept, y-intercept, increasing/decreasing intervals, end behavior, asymptote, point of discontinuity (hole) | | |

|  |  |  |
| --- | --- | --- |
| Unit: **REASONING WITH GEOMETRY**  Suggested Order: 6 of 8  Suggested Time: 15 days on a semester schedule (90-minute classes) | | |
| **Rationale**: This unit transitions into geometric concepts with an emphasis on reasoning, justification, and formalizing proof. Students will extend upon their work with proof in Math 2 (NC.M2.G.CO.9 and NC.M2.G.CO.10) focusing on both paragraph and flow proofs. Students are familiar with the properties of parallelograms from middle school and have categorized parallelograms and informally verified parallelogram properties through coordinate geometry in Math 1. Students will prove more theorems about triangles including the centers of triangles. This concept can be used as a transition into reasoning with circles. The Reasoning with Geometry Unit purposefully concludes with circles. In students’ work with circles, they will develop their understanding of radian measure through proportions in circles. This sets up a connection of circular motion to trigonometric functions in the next unit. | | |
| Overarching Standards - NC.M3.A-SSE.1b, NC.M3.A-SSE.2, NC.M3.A-REI.1, NC.M3.F-BF.1b | | |
| *Major Work* | | |
| Standards | Learning Intentions | Success Criteria |
| NC.M3.G-CO.11 NC.M3.G-CO.14  NC.M3.G-CO.10 | A. Construct logical arguments and explain reasoning with two-dimensional figures to prove geometric theorems about parallelograms and solve problems. Demonstrate an understanding of the properties of three of a triangle’s points of concurrency. | A1. I can use triangle congruency theorems and theorems about lines and angles to prove theorems about parallelograms including:   * Opposite sides of a parallelogram are congruent. * Opposite angles of a parallelogram are congruent. * Diagonals of a parallelogram bisect each other. * If the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle.   A2. I can write formal proofs (flow and paragraph) about properties of parallelograms.  A3. I can write formal proofs (flow and paragraph) to classify parallelograms given their properties.  A4. I can apply theorems about parallelograms to prove other theorems or to solve problems.  A5. I can explore and prove properties of centers (centroid, incenter, and circumcenter) of triangles.  A6. I can apply the properties of the centroid, incenter, and circumcenter of triangles to solve problems with and without contexts. |
| NC.M3.G-GPE.1  NC.M3.G-C.2  NC.M3.G-C.5  NC.M3.G-CO.14  NC.M3.G-MG.1 | B. Understand properties of circles and how to apply them algebraically and geometrically. Demonstrate understanding that within circles, segments, lines, and angles create special relationships and use these to solve geometric problems. | B1. I can use and apply the equation of a circle through completing the square to solve problems in modeling situations.  B2. I can describe and apply angle relationships (central, inscribed, and circumscribed) in a circle.  B3. I can describe and apply relationships between line segments and circles, including radii, diameter, secants, tangents, arcs, and chords.  B4. I can use similarity to demonstrate that the length of an arc, *s,* for a given central angle is proportional to the radius, *r,* of the circle.  B5. I can calculate arc lengths and areas of sectors of circles.  B6. I can define radian measure as the ratio of the length of the radius of the circle, *s/r*.  B7. I can use the connection between the intercepted arc, radius, and radian measure of an angle in a circle to solve problems with or without context.  B8. I can choose an appropriate geometric figure for modeling a particular real-world object, apply the properties, and solve problems by using appropriate geometric properties of the objects. |
| **Vocabulary:** Radius, diameter, center (of a circle), centroid, incenter, circumcenter, central angle, inscribed angle, circumscribed angle, tangent line, chord, secant line, radian (angle measure), arc length, sector. | | |

|  |  |  |
| --- | --- | --- |
| Unit: **TRIGONOMETRIC FUNCTIONS**  Suggested Order: 7 of 8  Suggested Time: 10 days on semester schedule (90-minute classes) | | |
| **Rationale**: This unit should immediately follow the Reasoning with Geometry unit. Students’ understanding of radians and the idea of circular motion are connections that can help students better understand trigonometric functions. | | |
| Overarching Standards - NC.M3.A-SSE.1b, NC.M3.A-SSE.2, NC.M3.A-REI.1, NC.M3.F-BF.1b | | |
| *Major Work* | | |
| Standards | Learning Intentions | Success Criteria |
| NC.M3.F-IF.1  NC.M3.F-IF.4  NC.M3.F-IF.7  NC.M3.F-IF.9 | A. Understand that triangular trigonometric functions are related to circular trigonometric functions in the coordinate plane. | A1. I can define a radian measure of an angle as the length of the arc on the unit circle subtended by the angle.  A2. I can explain how a ratio represents a value of a trig function for an angle.  A3. I can use key features to construct the graph of sine and cosine functions and interpret in context.  A4. I can state the amplitude, period, and midline of sine and cosine functions.  A5. I can use technology, graphs, and tables to compare sine graphs. |
| NC.M3.F-BF.3  NC.M3.F-TF.1  NC.M3.F-TF.2  NC.M3.F-TF.5 | B. Understand and interpret the key features, uses and limitations of multiple representations of trigonometric functions that model real world periodic behavior. | B1. I can describe the effect of a transformation on the graph of a sine function.  B2. I can explain that the radius of a circle is the ratio of the length of a subtended arc and its corresponding central angle.  B3. I can use technology to interpret the key features of sine graphs in a real world situation.  B4. I can work with angles in standard position to find coterminal and reference angles. |
| **Vocabulary:** Initial side of an angle, terminal side of an angle, ray, coterminal angles, sine, cosine, tangent, radian (angle measure), unit circle, domain, range, period, midline, amplitude, frequency, cycle, phase shift. | | |

|  |  |  |
| --- | --- | --- |
| Unit: **STATISTICS**  Suggested Order: 8 of 8  Suggested Time: 5 days on semester schedule (90-minute classes) | | |
| **Rationale**: This unit, Statistics, is more flexible in the pacing, than the other suggested units. Statistics can be taught as a stand-alone unit, since there is less integration and connection between standards. However, it is suggested that you do not break up the coherency of units that have intentionally been suggested to be taught in a certain order. (i.e - do not teach this unit between Reasoning with Geometry and Trigonometric Functions) | | |
| *Major Work* | | |
| Standards | Learning Intentions | Success Criteria |
| NC.M3.S-IC.1  NC.M3.S-IC.3 | A. Understand statistics as a process of making inferences about a population (parameter) based on results from a random sample (statistic).  Acknowledge the role of randomization in using sample surveys, experiments, and observational studies to collect data and understand the limitations of generalizing results to populations (related to randomization). | A1. I can distinguish between a sample (statistic) and a population (parameter).  A2. I can describe how to select a random sample from a given population.  A3. I can explain the purposes and the differences of sample surveys, observational studies, and experiments, including how randomization applies to each.  A4. I can distinguish between sample surveys, observational studies, and experiments.  A5. I can determine how results of a statistical study can be generalized to make conclusions about a population based on the sample. |
| NC.M3.S-IC.4  NC.M3.S-IC.5 | B. Understand simulation is useful for using data to make decisions. Know how to carry out a simulation with data for the purposes of: estimating population means or proportions, determining the margin of error for those estimates, and determining statistical significance.  Understand that samples can differ by chance. | B1. I can use data from a sample survey to estimate a population mean or proportion with a margin of error.  B2. I can determine and justify if results from an experiment are statistically significant.   * I can identify the parameter of interest in an experiment. * I can select and calculate sample statistics. * I can calculate the difference between the sample statistics. * I can set up and complete a simulation re-randomizing the groups. * I can compare the actual difference to the simulated differences to determine statistical significance.   B3. I can state a conclusion about the effectiveness or accuracy of a claim based on a sample. |
| NC.M3.S-IC.6 | C. Understand not all data that is reported is valid. Reports should be evaluated based on source, design of the study, and data displays. | C1. I can evaluate and make sense of a statistical article or website. |
| **Vocabulary:** Sample, population, margin of error, standard deviation, simple random sample, systematic sample, stratified random sample, cluster sample, self-selected sample, convenience sample, lurking variable, mean, standard deviation, survey, experiment, observation, simulation | | |