Homework 3.6: Natural Logs & Base e

Math 3

Name: _____

Directions: Expand each expression.

7) $\ln\left(\frac{7}{2}\right)^2$ $\quad 2\left(\ln 7 - \ln 2\right)$

$$\boxed{2\ln 7 - 2\ln 2}$$

8) $\log\dfrac{a^2}{b}$

$$\boxed{2\log a - \log b}$$

9) $\ln\dfrac{u^3}{wv^4}$

$$\boxed{3\ln(u) - \left(\ln w + 4\ln v\right)}$$

10) $\ln\left(5 \cdot 3^4 \cdot 11^6\right)$

$$\boxed{\ln 5 + 4\ln 3 + 6\ln 11}$$

Directions: Solve each equation for x. Check your answer.

17) $e^x = 2$

$\log_e 2 = x$

$\ln 2 = x$

$$\boxed{x = 0.6931}$$

18) $e^x = 59$

$\log_e 59 = x$

$\ln 59 = x$

$$\boxed{x = 4.0775}$$

19) $e^{-3n} = 83$

$\log_e 83 = -3n$

$\ln 83 = -3n$

$4.4188 = -3n$

$$\boxed{n = -1.4729}$$

20) $e^{k+7} = 26$

$\log_e 26 = k + 7$

$\ln 26 = k + 7$

$3.258 = k + 7$

$$\boxed{k = -3.7419}$$

25) $\ln(9r + 1) = \ln(r^2 + 9)$

$9r + 1 = r^2 + 9$

$r^2 - 9r + 8 = 0$

$(r - 8)(r - 1) = 0$

$$\boxed{r = 8 \quad r = 1}$$

26) $-8\ln -9x = -8$

$\ln(-9x) = 1$

$\log_e(-9x) = 1$

$e^1 = -9x$

$2.7183 = -9x$

$$\boxed{x = -0.3020}$$

27) $\ln(1 - 8n) - 10 = -7$

$\ln(1 - 8n) = 3$

$\log_e(1 - 8n) = 3$

$e^3 = 1 - 8n$

$20.0855 = 1 - 8n$

$19.0855 = -8n$

$$\boxed{n = -2.3857}$$

28) $-1 - 8\ln(-8b - 5) = 7$

$-8\ln(-8b - 5) = 8$

$\ln(-8b - 5) = -1$

$e^{-1} = -8b - 5$

$0.3679 = -8b - 5$

$5.3679 = -8b$

$$\boxed{b = -0.671}$$

Directions: Write and solve an equation for each of the following. Show all work. $y = Pe^{rt}$

1. $A = \$590.29, P = \$500.00, r = ???, n = continuous, t = 2$

$590.29 = 500\,e^{2r}$ $\log_e 1.18058 = 2r$ $r = 0.083$

$1.18058 = e^{2r}$ $\ln 1.18058 = 2r$ $\boxed{r = 8.3\%}$

$0.1660 = 2r$

2. $A = \$143.24, P = \$111.00, r = 5.1\%, n = continuous, t = ?$

$143.24 = 111\,e^{0.051t}$ $\ln 1.2905 = 0.051t$

$1.2905 = e^{0.051t}$ $0.2550 = 0.051t$

$\log_e 1.2905 = 0.051t$ $\boxed{t = 5\ \text{years}}$

3. $A = \$578.28, P = \$515.20, r = ???, n = continuous, t = 3.5$

$578.28 = 515.2\,e^{3.5r}$ $\ln 1.1224 = 3.5r$

$1.1224 = e^{3.5r}$ $0.1155 = 3.5r$ $\boxed{r = 3.3\%}$

$\log_e 1.1224 = 3.5r$ $r = 0.033$

4. A continuously compounded savings account had an initial deposit of \$10,000.00 and 10 years later has a balance of \$13,125.87. At what interest rate was the savings account?

$13125.87 = 10000\,e^{10r}$ $\ln 1.312587 = 10r$

$1.312587 = e^{10r}$ $0.272 = 10r$ $\boxed{r = 2.72\%}$

$\log_e 1.312587 = 10r$ $r = 0.0272$

5. \$250.00 is left in a savings account at 4.0% and the interest is compounded continuously. If the balance is now \$330.78, then how many years was the money been in the account?

$330.78 = 250\,e^{0.04t}$ $\ln 1.32312 = 0.04t$

$1.32312 = e^{0.04t}$ $0.28 = 0.04t$ $\boxed{7\ \text{years}}$

$\log_e 1.32312 = 0.04t$ $7 = t$

6. Hearing about the PlayStation 4 release 3.5 years ago, a teenager put his savings of \$500.00 into a continuously compounded savings account. He now has \$619.65. At what fixed rate was the interest?

$619.65 = 500\,e^{3.5r}$ $\ln 1.2393 = 3.5r$

$1.2393 = e^{3.5r}$ $0.2145 = 3.5r$

$\log_e 1.2393 = 3.5r$ $0.0613 = r$ $\boxed{r = 6.13\%}$