Math III Exponential/Geometric Series

Exponential functions are of the form $y = a(b)^x$, where *a* is the y-intercept or initial amount, and *b* is the growth/decay factor. If *b* is b > 1 it represents exponential growth and if $0 < b < 1$ it represents exponential decay.

Ex/ $f(x) = 5{,}236(1.08)^x$ *exponential growth, growth rate is 8%*
Ex/ $f(x) = 2{,}873(0.91)^x$ *exponential decay, decay rate is 9%*

Compounded interest uses the formula $A = p(1 + \frac{r}{n})^{nt}$, where *p* is the principle, *r* is the rate, *t* is the time, and *n* is the number of times the interest is compounded. (monthly *n* = 12, weekly *n* = 52, etc.)

Continuously compounded interest uses the formula $A = Pe^{rt}$, where *p* is the principle, *r* is the rate, and *t* is the time.

Mortgage Formula - monthly payment $= \frac{pi}{1-(1+i)^{-n}}$ where *p* is the principle, *n* is the number of total payments, *i* is the monthly interest rate.

Sum of finite geometric series is found by $S_n = \frac{a_1(1-r^n)}{1-r}$, where a_1 is the first term, *r* is the ratio (what each term is multiplied by to get to the next), *n* is term number the series stops.

Examples:

1. 288, -96, 32,... What is the approximate value of the sum of the 7th term?

$r = {}^-1/3$

$$S_7 = \frac{288(1-(^-1/3)^7)}{1-(^-1/3)} = 216.1$$

2. 360 + 480 + 640 + ... What is the approximate value of the sum of the 15th term?

$r = \frac{4}{3}$

$$S_{17} = \frac{360(1-(4/3)^{15})}{1-4/3} = 79737.4$$

3. What is the approximate value of the sum:

$r = -1/7$

$$8 - \frac{8}{7} + \frac{8}{49} - \cdots 8 \bullet \left(\frac{-1}{7}\right)^{2500} ?$$

$$S_{2500} = \frac{8(1-(^-1/7)^{2500})}{1-(^-1/7)} = 7$$

$n = 360$ months annual

4. Find the monthly payment of $175,000 home on a 30 year mortgage with a 3.5% interest rate.

$i = (1.035)^{1/12} = 1.\boxed{00287}$

$i = 0.00287$

↑

monthly

$$\text{payment} = \frac{175000(0.00287)}{1-(1.00287)^{-360}}$$

$$\text{payment} = \$780.37$$

5. Angela deposited $3000 into a savings account earning 4% interest compounded continuously, how much will she have after 6 years?

$y = Pe^{rt}$

$y = 3000e^{(0.04)(6)}$

$y = \$3813.75$

Hint: you can also graph your equation and find the intersections in y=

6. Sam deposited $5,500 into a savings account earning 5.6% interest compounded monthly. How many years had he been saving when the savings account has a balance of $8599.52?

$8599.52 = 5500\left(1 + \frac{0.056}{12}\right)^{12t}$

$1.5635 = (1.00467)^{12t}$

$\log 1.5635 = 12t \log 1.00467$

$t = \frac{\log 1.5635}{12 \log 1.00467}$

$t = 8$ years

7. Mary wants a dress that costs $450 for the prom. So far she has saved $275 and put it in a savings account for 1.5 years, what interest rate must she earn to have $450 by prom?

$y = Pe^{rt}$

$450 = 275e^{1.5r}$

$\ln 1.6364 = \cancel{\ln e}^{1.5r}$

$1.5r = \ln 1.6364$

$r = 0.328 = 32.8\%$

8. A board is made up of 9 squares. A certain number of pennies is placed in each square, following a geometric sequence. The first square has 1 penny, the second has 2 pennies, the third has 4 pennies, etc. When every square is filled, how many pennies will be used in total?
A. 521 B. 511 C. 256 D. 81

$r = 2$

$$S_9 = \frac{1(1-(2)^9)}{1-2} = 511$$

1	2	4
8	16	32
64	128	256