7.2 Quadratics & it's many forms

Vertex form: $y = a(x-h)^2 + k$

<u>vertex</u>: (h, k)

<u>min/max</u>: $a > 0$ min
$a < 0$ max

<u>line of symmetry</u>: $x = h$

<u>x-intercept</u>: substitute $y=0$
solve for x: use sq root method.

<u>y-intercept</u>: substitute $x=0$
use order of operations.

<u>Ex</u> $y = (x-4)^2 - 8$

<u>Ex</u> $y = -\frac{1}{2}(x+6)^2 - 10$

Ex) $y = 3(x+9)^2 - 18$ Ex) $y = \frac{2}{3}(x-7)^2 - 5$

Ex) $y = 2(x+12)^2 - 72$

7.3 Completing the Square:

Standard Form: $y=ax^2+bx+c$
Vertex Form: $y=a(x-h)^2+k$

① Put ( )'s around (ax^2+bx)
② Factor out "a" from inside the ( )'s
③ Take ½ of b; square it; put it inside ( )'s

$$y=a\left(x^2+bx+\underline{\left(\tfrac{b}{2}\right)^2}\right)+c-\underline{\left(\tfrac{b}{2}\right)^2\cdot a}$$

$$y=a\left(x+\tfrac{b}{2}\right)+\underline{c-\left(\tfrac{b}{2}\right)^2\cdot a}$$

④ multiply "a" by $\left(\frac{b}{2}\right)^2$; add opposite sign to "c"
⑤ Solve For: vertex, min/max, line of sym, x-int, y-int

Ex| $y=x^2-20x+19$

Ex| $y=x^2+14x+29$

Ex] $y=4x^2+24x-31$ Ex] $y=\frac{5}{6}x^2-35x+486$

7.1 & 7.2 Polynomial Degree & Finite Differences and Equivalent Quadratic Forms.

Writing Polynomial Eqns:

1. 1^{st} degree: Use $y - y_1 = m(x - x_1)$
2. 2^{nd} degree: use $y = ax^2 + bx + c$
   - step1: Substitute (3) x's & (3) corres y's
   - step2: Use $A^{-1}B$ to solve for a, b, and c.

Ex

x	1	2	3	4	5	6	7
y	-6	-10	-20	-36	-58	-86	-120

Ex

x	1	2	3	4	5	6
y	7	3	1	1	3	7

Ex

x	-4	-3	-2	-1	0	1	2	3	4	5
y	-30	-28.5	-26	-22.5	-18	-12.5	-6	1.5	10	19.5

Ex) -6 -10 -20 -36 -58 -86 -120

Ex) 7 3 1 1 3 7

Ex) -30 -28.5 -26 -22.5 -18 -12.5 -6 1.5 10 19.5

7.2 General/Standard Form: $y=ax^2+bx+c$

Vertex Form: $y=a(x-h)^2+k$

Factored: $y=a(x-p)(x-q)$

7.4 Solving Quadratics

Solve using:
1. Factor
2. square Root
3. Quadratic formula
4. Complete the Square

Ex\ $12x^2-4x=21$

Ex\ $2x^2-6x=176$

Ex\ $(x-18)^2=1521$

Evaluate: Find exact ans.

$$\frac{-8+\sqrt{(8)^2-4(2)(2)}}{2(2)}$$

Write in Factored form

Ex) $y = x^2 - 6x - 40$

Ex) $y = 4x^2 - 8x - 60$

Write quad eqn in general/standard form

1. $y = a(x - r_1)(x - r_2)$
2. substitute
3. find 'a' if necessary
4. multiply

Ex $a = 3$ x-int: -6 and 5

Ex x-int: 10 and -4 y-int: 6

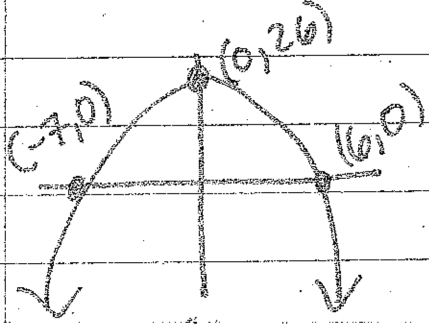
Ex x-int: 2.7 y-int: -3.24

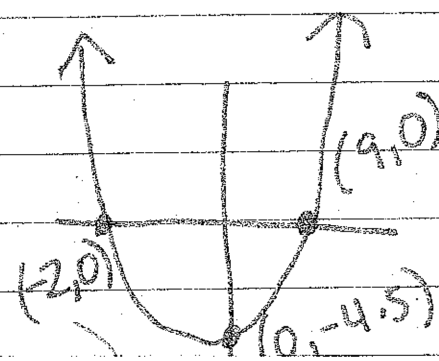
7.6 Factoring Polynomials

Find x + y intercepts

Ex) $y = (x-9)(x+8)$ Ex) $y = -(x+13)^2$ Ex) $y = .45(x-10)(x+115)$

Write the factored form of the quad function
Vertical scale factor: Use matrix eqn. to find a.





Product of factors

Ex x^2-19 Ex x^2+22 Ex x^4-17x^2+16

Ex $x^3+2x^2-13x+10$ (use calc)

Sketch

a) quad function no real zero, axis of sym. $x=-3$

b) cubic function, two real zeros and a positive y-int.

Long Division: use when divisor has an exponent or the lead term has a coefficent other than 1.

1. set up division: fill in missing terms with zeros.
2. identify the controller: lead term on divisor
3. divide: distribute over the divisor
4. Subtract: change to addition distribute the neg. through
5. Repeat
6. Write answer.

Synthetic division $(x \pm C)$

1. set up
2. change sign on C #
3. put coeffiecents of poly. on top row. fill in missing terms.
4. Drop Lead
5. multiply and add
6. Write answer one degree lower than original problem.

$$x^2+2x-5\,\overline{\big)\,4x^5+8x^4-11x^3+12x^2-61x+32}$$

$$8x-13\,\overline{\big)\,48x^5-54x^4-95x^3+107x^2-66x+65}$$

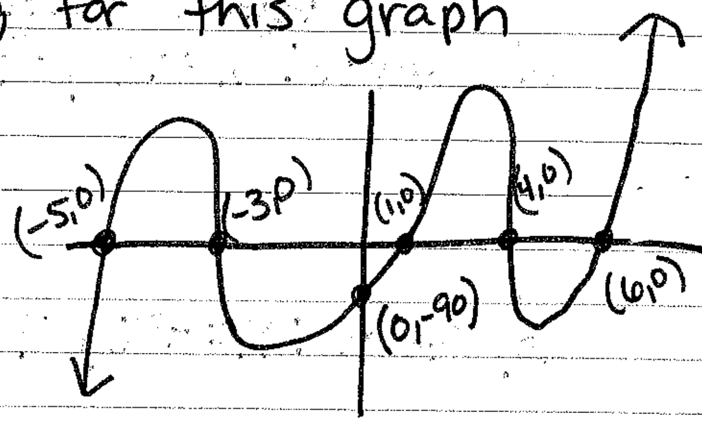
$(11x^4 - 8x^2 + 7) \div (x-2)$

$(3x^5 + 4x^4 - 6x + 15) \div (x+4)$

7.7 Higher-Degree Polynomials

Answer the following for this graph

a) Identify the zeros
b) Find the y-int
c) What is the lowest possible degree?
d) Write the equation in factored form.



Write a polynomial with these features:

a) a linear function whose graph has x-int of 4.

b) A quad function whose graph has only one x-int, 4.

c) a cubic function whose graph has only one x-int, 4.

Write the lowest-degree poly with the given info, then give the lowest degree possible.

a) zeros: $x = 1/3, -2/5, 0$ y-int: 0

b) zeros: $x = -5i, -1$ (triple root), 4   y-int: -100

Worksheet

9.4 Notes

To find remainder $P(x) \div (x-c)$

① Divide

or

② Evaluate $P(c)$

Ex) $(x^4+5x^3-5x^2+45x-126)$ at $x=3$
$x=-7$

Find the missing factors: Divide
Calc

Ex) $x^3+13x^2+34x-48=(x+6)(\ ?\ )(\ ?\ )$

Ex) $5x^3-58x^2+111x+54=(x-9)(\ ?\ )(\ ?\ )$

$V=6x^3+49x^2-106x+40$ find the width + height if the length $=x+10$

List of possible <u>Rational Roots</u> (Calc table)

1) put equation in descending order
2) p # last number or coefficient on last term
3) q # coefficent on lead term
4) List factors of p # use ±
5) List factors of q #
6) List of possible rational zeros (roots)
   $\pm p/q$

<u>Ex</u> $f(x) = x^5 - 7x^4 + 8x^3 - 5x^2 + 12x - 32$
(leave 5 lines)

<u>Ex</u> $g(x) = 3x^4 + 10x^3 - 14x + 10$
(leave 5 lines)

<u>Ex</u> $h(x) = 8x^3 + 3x^2 - 18x - 24$
(leave 5 lines)

Solve higher (greater than 2) order polynomials

1. make list
2. put eq in Y_1
3. table set
   Start: 0
   ΔTBL: $1/a$ or $1/q$
4. Table: find all the roots you can
   a) If Roots = Degree, you're finished
   b) If Roots < Degree, synthetically divide until you have a 2nd degree equation.
      1) quad formula
      2) complete square
      3) sq root property

Ex| $f(x) = x^3 - 6x^2 + 5x + 12$

Ex| $g(x) = x^3 - 5x^2 + 9x - 45$

Ex| $g(x) = 12x^5 - 67x^4 + 39x^3 + 96x^2 + 2x - 12$

Ex| $h(x) = 6x^3 + 17x^2 + 6x - 8$

Ex| $f(x) = x^4 - 21x^2 - 100$

9.4 Worksheet Directions

1-6 → Use the best choice:
Synthetic or long
Write your ans. like normal

7-9 → Just write the final ans
from the syn. division.

10-13 → Synthetic division

14-17 → Plug in the # and give ans. for
$f(x)$.

18-21 → Use synthetic for the first zero,
the use trinomial factoring.

22-25 → follow directions

9.6

Number of Solutions = Degree

Ex] $x^4+5x^3-7x^2+10x=19$

1-4 Ex] $-8x^5+13x^3+6x-2=0$

To determine if x is a zero

① Synthetically divide: zero remainder

or

② Remainder Theorem: Substitute for x: evaluate

5-8 Ex] $f(x)=x^4-2x^3+x^2-32x-240,\ x=-4i$

Ex] $g(x)=x^4+7x^3-6x^2+14x-16,\ x=2$

Zeros → factors ① x=

② move across =

9-12 Ex] $-6, 9, -3$

Ex] $0, 4, 6i, -6i$

Write a polynomial function (Standard form) from zeros

13-16 ① x=

② move across =

③ multiply (always multiply conjugates)

Ex $-5, 1, 6$ Ex $0, 3, 4i, -4i$

x-int → factors: Same as zeros → factors

17-20 Ex $(-7,0)(1,0)(4,0)$ Ex $(0,0)(2,0)(-8,0)$

Polynomial function given x-int: same as from zeros

21-24 Ex $(-9,0)(0,0)$ Ex $(-4,0)(1,0)(4,0)$

Write polynomial function as product of linear (1^{st} degree) factors.

25-30
1. calc: roots
2. syn' divide
3. factor: or solve
4. write as factors

Ex $x^3-2x^2+25x-50$ Ex $3x^4-5x^3+11x^2-15x+6$

Irrational and Complex Solutions come in conjugate pairs.

If $3+7\sqrt{21}$ is a solution, then ______ is a solution.

If $2-5i$ is a solution, then ______ is a solution.

If 7 is a solution, then is -7 a solution?

Find the missing zeros

Ex| 6^{th} degree, zeros at $20, -3, 4+9i, -5-7\sqrt{2}$

Ex| 3^{rd} degree, zeros at $6-i, 8$

List all possible rational roots

Ex| $f(x)=6x^4+11x^2-24$

Ex| $g(x)=x^9-3x^7+5x^3-50$

Find all the Zeroes

Ex$_1$) $f(x) = 4x^5 - 43x^4 + 61x^3 + 290x^2 - 92x - 40$

Ex$_2$) $g(x) = 2x^3 - 11x^2 + 174x + 90$