Notes on Matrices
6.1

Definition of a Matrix – A rectangular (or square) array of numbers, can be written using brackets or parentheses.

Element – One of the entries in a matrix. The address of an element is given by listing the row number then the column number.

**A matrix can be named using its dimensions.

Dimension – The number of Rows and Columns of a matrix, written in the form Row x Column.

Examples:

1. $A = \begin{bmatrix} 2 & -1 \\ 0 & 5 \\ -4 & 8 \end{bmatrix}$ 2. $B = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$ 3. $C = \begin{bmatrix} 0 & 5 & 3 & -1 \\ -2 & 0 & 9 & 6 \end{bmatrix}$

Row Matrix - A horizontal set of numbers in a matrix

Column Matrix – A vertical set of numbers in a matrix

Square Matrix – A matrix with equal numbers of rows and columns.

Using matrices to solve problems:

Jim, Mario and Mike are married to Shana, Kelly and Lisa. Mario is Kelly's brother and lives in Florida with his wife. Mike is shorter than Lisa's husband. Mike works at a bank. Shana and her husband live in Kentucky. Kelly and her husband work in a candy store. Who is married to whom? Find out using a matrix!

Equal Matrices

Solve for x and y.

1. $\begin{bmatrix} 2x \\ 2x+3y \end{bmatrix} = \begin{bmatrix} y \\ 12 \end{bmatrix}$

2. $\begin{bmatrix} 3x+y \\ x-2y \end{bmatrix} = \begin{bmatrix} x+3 \\ y-2 \end{bmatrix}$

3. $\begin{bmatrix} 2x & 3 & 3z \end{bmatrix} = \begin{bmatrix} 5 & 3y & 9 \end{bmatrix}$

6.2

Adding and Subtracting Matrices

Only matrices with ______________________________ can be added or subtracted.

The resulting matrix has ___________________ dimensions.

Examples

1. $\begin{bmatrix} -2 & 0 & 4 \\ 3 & -10 & 12 \\ 3 & -2 & -2 \end{bmatrix} + \begin{bmatrix} -4 & 6 & 0 \\ -15 & 2 & -4 \\ 6 & 7 & 1 \end{bmatrix}$

2. $\begin{bmatrix} 3 & -4 \\ 0 & 2 \end{bmatrix} + \begin{bmatrix} 4 & 6 & 8 \\ 0 & 2 & 4 \end{bmatrix}$

3. $\begin{bmatrix} 2 & -18 \\ 20 & -5 \end{bmatrix} - \begin{bmatrix} -4 & 2 \\ -5 & 1 \end{bmatrix}$

4. $\begin{bmatrix} -3 & 10 & 2 \\ -10 & 8 & -6 \\ 0 & 1 & 0 \end{bmatrix} - \begin{bmatrix} 5 & 4 & 3 \\ 2 & 1 & 0 \\ -8 & -10 & -4 \end{bmatrix}$

6.3

Scalar Multiplication

Examples:

1. $-2\begin{bmatrix} 7 & -1 & 0 \end{bmatrix}$

2. $4\begin{bmatrix} -2 & 0 \\ 4 & -5 \end{bmatrix}$

3. $-5\begin{bmatrix} 1 & 2 & -3 \\ 4 & -5 & 6 \\ -7 & 8 & -9 \end{bmatrix}$

Matrix Multiplication

Multiply rows times columns
**You can only multiply if the number of columns in the 1st matrix is equal to the number of rows in the 2nd matrix.

1. $\begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 3 & -9 & 2 \\ 5 & 7 & -6 \end{bmatrix}$

2. $\begin{bmatrix} 3 & -9 & 2 \\ 5 & 7 & 6 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$

3. $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 4 & -3 \\ -2 & 5 \end{bmatrix}$

Transformations

Pre-image Matrix: A matrix with the coordinates of each point arranged with x's on the top row and y's on the bottom row. (original shape)

Image Matrix: A matrix of the coordinates after a transformation.

Translation Matrix: A matrix that matches the shape of the matrix of the image. The top row is the 'x' movement, while the bottom row is the 'y' movement.

Reflection over the x-axis = $\begin{bmatrix} & \\ & \end{bmatrix}$ **y-axis** = $\begin{bmatrix} & \\ & \end{bmatrix}$

Rotation (clockwise): **0°** $\begin{bmatrix} & \\ & \end{bmatrix}$ **90°** $\begin{bmatrix} & \\ & \end{bmatrix}$ **180°** $\begin{bmatrix} & \\ & \end{bmatrix}$ **270°** $\begin{bmatrix} & \\ & \end{bmatrix}$

6.4

Determinants

Determinant of a 2x2 matrix: - Multiply elements in the forward diagonal and then subtract the multiplied elements in the backward diagonal.

Find the determinant of each:

1. $\begin{vmatrix} -5 & -7 \\ 11 & 8 \end{vmatrix}$

2. $\begin{vmatrix} 3 & 2 \\ -1 & 5 \end{vmatrix}$

3. $\begin{vmatrix} 10 & -2 \\ 0 & -3 \end{vmatrix}$

**To find a determinant you must have a _________ matrix!!

Determinant for a 3x3 matrix: Diagonal method
* the determinant can be computed by adding the product of the forward diagonals and subtracting the product of the backward diagonals.

Examples:

1. $\begin{vmatrix} -2 & 3 & 8 \\ 6 & 7 & -1 \\ -4 & 5 & 9 \end{vmatrix}$

2. $\begin{vmatrix} 5 & -1 & 2 \\ 2 & -3 & 5 \\ 3 & 2 & -3 \end{vmatrix}$

3. $\begin{vmatrix} -1 & 0 & 4 \\ 2 & -2 & 2 \\ 3 & 0 & -1 \end{vmatrix}$

Determinant for 3x3 matrix: Calculator Method

1. $\begin{vmatrix} 1 & 2 & 3 \\ 0 & -4 & 1 \\ 0 & 3 & -1 \end{vmatrix}$

2. $\begin{vmatrix} 5 & -2 & 1 \\ 0 & 3 & -1 \\ 2 & 0 & 7 \end{vmatrix}$

6.5

Identity and Inverse Matrices

Identity Matrix $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ A square matrix with 1's on the main diagonal and 0's every where else

Inverse of a 2x2 matrix $[A][B] = [B][A] = I_n$

This is called "invertible"

Original matrix: $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ To find inverse: $\frac{1}{det}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

Examples:

1. $\begin{bmatrix} 5 & 3 \\ 2 & 1 \end{bmatrix}$

2. $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

3. $\begin{bmatrix} 2 & -5 \\ 0 & 7 \end{bmatrix}$

???Is there ever a square matrix that does not have an inverse???

Coding

Encoding: converting information from a source into symbols for communication or storage.

Decoding: is the reverse process of encoding, making it possible for someone to understand

Encoding:

The message, MRS WEAVIL IS SUPER AWESOME, is to be encoded using the matrix A = $\begin{bmatrix} 5 & -3 \\ 2 & -1 \end{bmatrix}$

Convert the message into 1 X 2 uncoded row matrices

Multiply each by the matrix A from above, and write the message in code.

Decoding:

Use the inverse of A = $\begin{bmatrix} 6 & -2 \\ 3 & -4 \end{bmatrix}$ **to decode the message below**

81 -30 144 -72 27 -36 114 -38 96 -86 33 -14 120 -40

****[A] can be and will be different for every problem. The two above are just random examples.**

6.6

Solving Systems of equations using matrices

- Coefficient Matrix – The matrix formed by the coefficients in a linear system.
- Variable Matrix – The matrix formed by the variables in a linear system.
- Constant Matrix – The matrix formed by the constants in a linear system.

Example: $\begin{bmatrix} 3 & -2 \\ 4 & 1 \end{bmatrix} \bullet \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ 8 \end{bmatrix}$

Steps to solving:

1. Inverse of [A] =

2. Multiply =

3. Simplify =

Example: Solve using the inverse matrix method in your calculator

$4x - 12y = 7$
$x + 6y = 9$