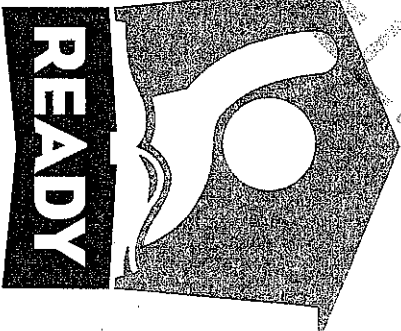


Student Name:

KEY

Spring 2013  
North Carolina  
Measures of Student Learning:  
NC's Common Exams  
**Advanced Functions  
and Modeling**



Public Schools of North Carolina  
State Board of Education  
Department of Public Instruction  
Raleigh, North Carolina 27699-6314

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# Student Booklet



- 1 The table below shows the probability distribution of the number of televisions in each house in a community.

Televisions	Probability
0	0.04
1	0.38
2	0.27
3	$x$
4	$y$
5 or more	0.13

at least 3 TVs

What is the probability that a house in the community will have at least 3 televisions? The probabilities must add to 1:

A 0.69

B 0.31

C 0.18

D 0.09

$$0.04 + 0.38 + 0.27 + x + y + 0.13 = 1 \quad 10/30 \text{ (B-2)}$$

P(at least 3)

$$x + y + 0.13 = 0.31$$

- 2 Anna and Zach each have \$600 to invest. Anna's investments earn a rate of 10.5%, and Zach's investments earn a rate of 6.5%. **Approximately**, how much more money will Anna have than Zach when Zach's investments are worth \$900? (Assume continuous compounding.)

A \$184

B \$241

C \$255

D \$264

Use  $A = Pe^{rt}$  for  $2/20$  (C-9)  
continuous compounding.

① Find time it will take Zach to have \$900;  
 $900 = 600e^{.065t}$  (Solve by graphing or w/ logs)

$$t = 6.24 \text{ years}$$

② Plug 6.24 years into the Anna equation:  
 $A = 600e^{.105(6.24)} = \$1155.00$

Go to the next page.

go to [hennesseymath.com](http://hennesseymath.com) and review posts for the listed dates.

③ Subtract:

$$\$1155 - 900 = \$255$$



3 A solution's pH is given by the function  $p(t) = -\log(t)$ , where  $t$  is the hydronium ion concentration, in moles per liter. A sample of coffee has a pH of 5.0. What is the approximate hydronium ion concentration of the sample?

- A 0.00001  $-\log(t) = 5 \quad 3/18 \quad (C-21) \quad t = 10^n \text{ concentration}$
- B 0.00001  $\log(t) = -5 \quad P = \text{pH (acidity)}$
- C 0.0001  $\log(t) = -5$
- D 0.001  $\text{Remember the } 3/12 \quad (C-18)$

4 A sequence is shown below.

1, 0.1, 0.01, 0.001, 0.0001

What is the sum of the sequence?

- A  $1\frac{1}{10}$
- B  $1\frac{1}{9}$   $3/20$
- C  $1\frac{2}{9}$   $4/8$
- D  $1\frac{9}{10}$

This is a geometric series, so you could use the formula  $S = \frac{a_1}{1-r}$ , where  $a_1 = 1$  and  $r = .01$ .

But since this is multiple choice you could add the first 5 terms and get 1.1111, which is close to  $1 + \frac{1}{9} = 1.1$ .

5 Which statement is true about the sequence shown below?

0, 4, 5, 12, 22.5, ...  $3/20, 4/8$

- A The series converges because the limit of the sequence as  $n$  approaches infinity is infinity.  $\leftarrow$  makes no sense
- B The series converges because the limit of the sequence as  $n$  approaches infinity is 30.  $\leftarrow$  only true if the sequence is geometric and
- C The series diverges because the limit of the sequence as  $n$  approaches infinity is infinity.  $|r| < 1$ .
- D The series diverges because the limit of the sequence as  $n$  approaches infinity is 30.  $\leftarrow$  makes no sense

6 A pharmaceutical company is creating a new cholesterol drug to prevent heart disease. The company must collect data by testing the drug before it is approved. Which would be the best method of data collection?

- A experimental study
- B observational study
- C simulation
- D survey

Only an experimental study can determine if the drug is causing disease to be prevented.



7 The table below shows the midterm and final exam grades of ten students.

	Midterm	68	78	92	90	88	82	94	83	71	62
	Final Exam	62	77	99	87	85	84	95	98	72	64

Which comparison between the midterm grades and the final exam grades is true? *handbook*

**A** The final exam grades have a higher mean and standard deviation than the midterm grades.

**B** The final exam grades have a lower mean and standard deviation than the midterm grades.

**C** The final exam grades have a higher mean and a lower standard deviation than the midterm grades.

**D** The final exam grades have a lower mean and a higher standard deviation than the midterm grades.

Enter data into L1 and L2, hit STAT → Calc → 1-Var Stats.

8 A baseball team scored the following number of runs in its games this season: 6, 2, 5, 9, 11, 4, 5, 8, 6, 7, 5. There is one more game in the season. If the team wants to end the season with an average of at least 6 runs per game, what is the least number of runs the team must score in the final game of the season?

*9/110 (scale handbook)*

*Enter the data into L1, with*

*choice A is the last number.*

*Calculate the mean (see #7). If*

*it's less than 6, try choice B.*

*Keep going until the mean*

*reaches 6.*

9 If the probability of giving birth to a boy is 0.52, what is the approximate probability of giving birth to four consecutive boys?

*10/8 Multiplication Rule*

*10/15 P(B,B,B,B) =*

*P(B)\*P(B)\*P(B)\*P(B)*

*= 0.52^4 =*

- A 0.021
- B 0.062
- C 0.073**
- D 0.130

10 How many more ways can 10 juniors running for the positions of president, vice president, secretary, and treasurer be selected when compared to 12 sophomores running for 5 identical positions of class representative?

*Count permutations of*

*10/21*

*11,628 juniors (order matters):*

*10P4 = 5040*

*10/30 (6-3)*

*4,248 Count combinations of*

*sophomores: 12C5 = 792*

*Subtract: 5040 - 792 = 4248*

*A starting line for a hockey team should consist of 3 offensive players, 2 defensive players, and 1 goaltender. A coach has 11 offensive players, 6 defensive players, and 2 goaltenders from which to choose the starting line. How many unique starting lines can the coach create?*

**A 132** *Calculate how many*

**B 792** *combinations of each*

**C 4,950** *type of player are possible:*

**D 59,400**

*6C2 = 15*

*2C1 = 2*

*Use the Fundamental Counting Principle*

*Think tree diagrams and the Friendlys*

*Problem) and multiply the choices:*

*10S \* 1S \* 2 = 4950*





- 12 It costs a bakery \$3.50 to make apple pies that sell for \$12 the first day they are baked.
- If a pie is not sold on the first day, the new price is \$8.50.
  - The probability of selling the apple pie the first day is 75%.
  - There is a 12% probability of selling it on the second day.
  - If the apple pie does not sell by the end of the second day, it is donated.

What is the **approximate** expected profit per pie for the bakery on the sale of its apple pies? **1** Make a probability distribution (see #10):

	Profit	Probability
A	\$5.67	
B	\$6.52	0.75
C	\$9.57	0.12
D	\$10.02	0.13

add to 1.  
 - \$3.50  
 - \$3.50 \* .13 = \$0.52

- 13 The number of household members,  $x$ , living in Cityville homes has the following probability distribution:

$x$	1	2	3	4	5	6	7	8
$P(x)$	0.21	0.28	0.16	0.22	0.06	0.04	0.02	0.01

What is the expected size of a household in Cityville?

- A 2.43  
 B 2.89  
 C 3.17  
 D 4.50
- Calculate the expected value:  
 $1 * .21 + 2 * .28 + 3 * .16 + 4 * .22 + 5 * .06 + 6 * .04 + 7 * .02 + 8 * .01$   
 $= 2.89$

- 14 What is the middle term for the expansion of  $(x^2 + 3)^{12}$ ?
- 7th term
- A  $729x^{12}$   
 B  $924x^{12}$   
 C  $673,596x^{12}$   
 D  $665,280x^{12}$
- $10/29 = 924x^{12} \cdot 729$   
 $10/30 = 673,596x^{12}$

- 15 Abby took an 8-question multiple-choice quiz. Suppose that her probability of correctly answering any question is 0.75. What is Abby's probability of incorrectly answering exactly two questions on the quiz?
- A  $P = 0.089$   
 B  $P = 0.240$   
 C  $P = 0.311$   
 D  $P = 0.623$
- 1 Find how many combos of wrong answers she could get:  
 $8C2 = 28$   
 $10/15 (0.22)$   
 $10/30 (0.3)$   
 $= .28 * .75^6 = .0111$

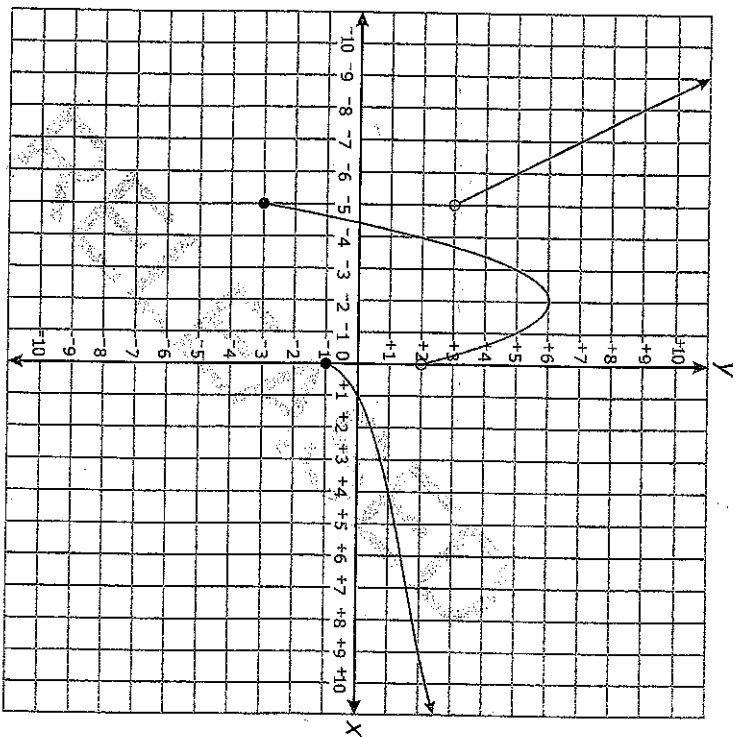
- 16 Which function results by shifting the graph of  $y = \ln(x + 3) - 6$  to the left 4 units and down 3 units?
- A  $y = \ln(x + 7) - 9$   
 B  $y = \ln(x - 1) - 9$   
 C  $y = \ln(x + 7) - 3$   
 D  $y = \ln(x - 1) - 3$

11/20  $y = \ln(x+4+3) - 6 - 3$   
 12/3  $= \ln(x+7) - 9$   
 12/6

left 4: add 4 to the input  $x$   
 down 3: subtract 3 from the output,  $\ln(x+3) - 6$



17 Which piecewise function is graphed below?



A  $f(x) = \begin{cases} -2x - 7 & \text{for } x < -5 \\ -(x+2)^2 + 6 & \text{for } -5 \leq x < 0 \\ \sqrt{x} - 1 & \text{for } x \geq 0 \end{cases}$  11/26

B  $f(x) = \begin{cases} -2x - 7 & \text{for } x < -5 \\ -(x-2)^2 + 6 & \text{for } -5 \leq x < 0 \\ \sqrt{x} - 1 & \text{for } x \geq 0 \end{cases}$  12/3

C  $f(x) = \begin{cases} -2x - 7 & \text{for } x < -5 \\ -(x+2)^2 + 6 & \text{for } -5 \leq x < 0 \\ \sqrt{x} - 1 & \text{for } x \geq 0 \end{cases}$  12/4

D  $f(x) = \begin{cases} -2x - 7 & \text{for } x \leq -5 \\ -(x+2)^2 + 6 & \text{for } -5 < x \leq 0 \\ \sqrt{x} - 1 & \text{for } x > 0 \end{cases}$  12/6

18 A function,  $f(x)$ , is shown below.

$$f(x) = \begin{cases} x - 4 & \text{for } 0 \leq x < 2 \\ x^2 - 3x + 4 & \text{for } 2 \leq x < 4 \\ 5 & \text{for } 4 \leq x < 7 \end{cases}$$

What is the range of  $f(x)$ ?

- A [4, 5]
- B [4, 8]
- C [4, 2)  $\cup$  [2, 5]
- D [4, 2)  $\cup$  [2, 8]

the parabola must be shifted 2 left ( $= x+2$ ), not 2 right.

$x=0$  must be included in the 3rd piece of the function.

12/4  
12/6

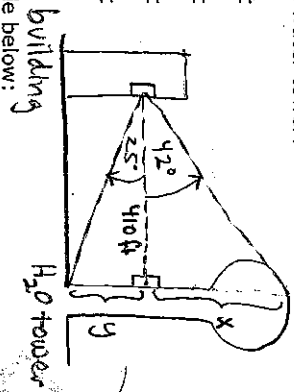
Therefore  $-4 \leq y < -2$  OR  $2 \leq y < 8$ .  
So we write  $[-4, -2) \cup [2, 8)$

Answer choices are on the following page.



19 A water tower is located 410 feet from a building. From a window in the building, it is observed that the angle of elevation to the top of the tower is 42 degrees and the angle of depression to the bottom of the tower is 25 degrees. **Approximately** how tall is the water tower?

- A 191 feet
- B 369 feet
- C 448 feet
- D 560 feet

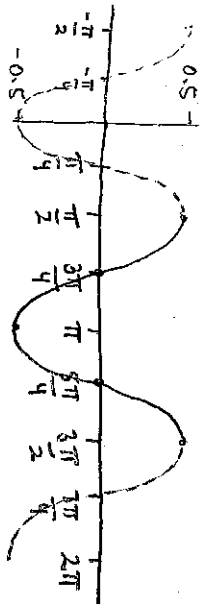


x	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	$\pi$	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$
y	0.5	0	-0.5	0	0.5

4/21  
4/22  
4/29 (D-14 Part 1)

Which function fits the data?

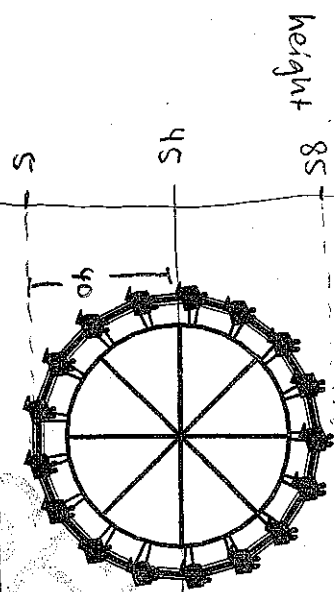
- A  $y = 0.5 \cos(2x - \pi) = 0.5 \cos(2x - \frac{\pi}{2})$  12/6
- B  $y = 0.5 \cos(x - \pi)$  ← frequency mod 2, bc 2. S/17
- C  $y = 0.5 \cos(2x + \frac{\pi}{2})$  S/14
- D  $y = \cos(2x + \frac{\pi}{2})$  ← amplitude must be 0.5



OR: Make a scatterplot of the data, then check each function to see if it fits. The mode should be in radians.



21 A Ferris wheel has a diameter of 80 feet. Riders enter the Ferris wheel at its lowest point, 5 feet above the ground, at time  $t = 0$  seconds. One complete rotation takes 65 seconds.



Which function models a rider's vertical height,  $h(t)$ , at  $t$  seconds?

- A  $h(t) = -80 \cos(\frac{2\pi}{65}t) + 5$  S/7
- B  $h(t) = -40 \cos(\frac{2\pi}{65}t) + 45$  S/14
- C  $h(t) = -45 \cos(\frac{65}{2\pi}t) + 40$
- D  $h(t) = -5 \cos(\frac{65}{2\pi}t) + 80$

vertical shift of 45.

axis of oscillation has moved up 45 units.

CHECK YOUR MODE (degrees)

USE RIGHT TRIANGLE TO FIND X AND Y:

$\tan 42^\circ = \frac{x}{410}$

$410 \tan 42^\circ = x$

$369.17 = x$

$\tan 25^\circ = \frac{y}{410}$

$410 \tan 25^\circ = y$

$y = 191.14$

Add x and y to get the total height of the tower:

$x + y \approx 560 \text{ ft.}$

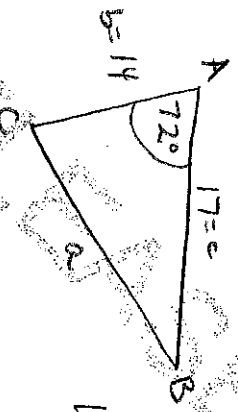


22 How does the graph of  $g(x) = 0.5\cos(2x)$  differ from the graph of its parent function,  $f(x) = \cos(x)$ , over the interval  $-\pi \leq x \leq \pi$ ?

- A The amplitude is smaller, and the period is shorter.  $S/17$
- B The amplitude is smaller, and the period is longer.  $S/14$
- C The amplitude is larger, and the period is shorter.  $\checkmark$  The frequency has increased from 1 to 2.
- D The amplitude is larger, and the period is longer.  $\checkmark$  The higher the frequency, the shorter the period.

23 Two sides of a triangle measure 14 ft and 17 ft, respectively. Approximately how long is the third side of the triangle?

- A 18.4 ft
- B 20.3 ft
- C 25.1 ft
- D 30.7 ft



Law of Cosines:  
 $a^2 = 14^2 + 17^2 - 2 \cdot 14 \cdot 17 \cdot \cos 72^\circ$   
 $a \approx 19.4$   
 $\approx 19$

24 In a geometric sequence,  $a_1 = 12$  and  $r = \sqrt{2}$ . What is the approximate sum of the first 20 terms of the sequence?

- A 339.4
- B 8,688.9
- C 29,624.9
- D 29,636.9

$3/26$ ,  $4/8$

$$S_{20} = \frac{12(1 - \sqrt{2}^{20})}{(1 - \sqrt{2})} = 29,636.89$$

25 A bathroom floor has tiles arranged in 9 circles. The innermost circle contains 9 tiles. Each successive circle contains 9 more tiles than the previous circle. How many total tiles are on the bathroom floor?

- A 81
- B 396
- C 405
- D 729

Arithmetic Series:  
 $4/2$  Find  $a_9$ :  
 $a_n = a_1 + (n-1)d$   
 $a_9 = 9 + (9-1) \cdot 9 = 81$

This is the end of the multiple-choice portion of the test.

② Find  $S_9$   
 $S_n = \frac{n}{2}(a_1 + a_n)$   
 $S_9 = \frac{9}{2}(9 + 81)$   
 $S_9 = 405$